Effective field theories for heavy Majorana neutrinos

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based on arXiv 1307.7680 and TUM-EFT 43-13 (work in progress)

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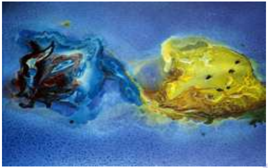


Outline

Open problem in Particle-Cosmology

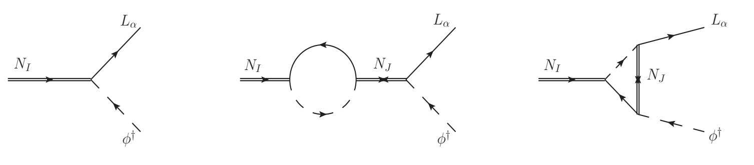
Baryon asymmetry in the Universe

- universe strongly matter-antimatter asymmetric

$$\eta_B = \frac{n_{\mathrm{B}} - n_{\bar{\mathrm{B}}}}{n_\gamma} = (6.2 \pm 0.15) \times 10^{-10}$$

E. Komatsu et al. WMAP collaboration

- within SM: $\eta_B \sim 10^{-18}$
- Baryogenesis via Leptogenesis: $\Delta L \to \Delta B$



- Three Sakarov conditions: $\Delta L \neq 0$, C and CP violation, out-of-equilibrium
- Massive Majorana neutrino play the relevant role

Strong Wash-out

- The out-of-equilibrium provided by H (universe expansion)

Decay parameter for the wash-out

$$K_1 = \frac{\Gamma_1^{(T=0)}}{H(T=M_1)} = \frac{M_1\left(F^\dagger F\right)_{11}}{8\pi 1.66\sqrt{g^*}\frac{M_1^2}{M_{Pl}}} = \frac{\tilde{m}_1}{m_*}$$

where we may define

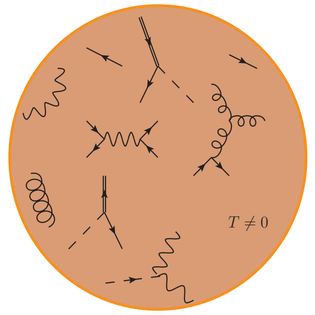
- $\tilde{m} = \left(F^\dagger F\right)_{11}\frac{v^2}{M_1}$, effective neutrino mass
- $m_* = 8\pi\, 1.66\sqrt{g^*}\frac{v^2}{M_{Pl}} \simeq 1.1\times 10^{-3}$ eV *Cfr. P. di Bari and M. Garny*

- Because $\tilde{m} \simeq m_{\rm sol} = \sqrt{\Delta m^2_{\rm sol}} \simeq 0.009 \Rightarrow K > 1$

1) Majorana neutrinos tracks **almost** the equilibrium distribution
2) The final lepton asymmetry is produced in a non-relativistic regime

Non-relativistic neutrinos in medium

Interaction at $T \neq 0$



1) Thermal production rate

A. Salvio, P. Lodone and A. Strumia (2011)
M. Laine and Y. Schroder (2012)

$$\Gamma^{(T=0)} \to \Gamma(T)$$

1) CP asymmetry at finite T

M. Garny, A. Hohenegger and A. Kartavtsev (2010)

$$\epsilon = \frac{\Gamma_\ell - \Gamma_{\bar{\ell}}}{\Gamma_\ell + \Gamma_{\bar{\ell}}} \to \epsilon(T)$$

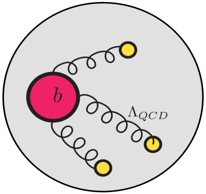
The non relativistic has been already explored in many aspects

D. Bödeker, M. Wörmann (2014)

- the hierarchy of scales $M \gg T$ has been not fully exploited
- we want to investigate the effective field theory (EFT) approach

Heavy particle Lagrangian

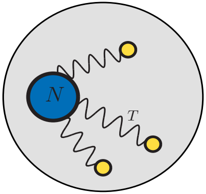
Heavy Quark Effective Theory



- $p^\mu = M v^\mu + k^\mu$ with $|k| \sim \Lambda_{QCD} \ll M$
- $i \frac{\not{p}+M}{p^2-M^2+i\epsilon} \to \left(\frac{1+\not{v}}{2}\right) \frac{i}{v\cdot k+i\epsilon}$

$$\mathcal{L}_{\rm HQEFT} = \bar{h}\,(iv\cdot D)\,h + \sum_i c_n\left(\frac{\mu}{M}\right)\frac{\mathcal{O}_n(\mu,\Lambda_{\rm QCD})}{M^{d_n-4}} + \mathcal{L}_{light}$$

Heavy Majorana neutrinos



- $p^\mu = M v^\mu + k^\mu$ with $|k| \sim T \ll M$

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2}\bar{\psi}\,(i\not{\partial} - M)\,\psi - F_f \bar{L}_f \tilde{\phi} P_R \psi - F_f \bar{\psi} P_L \tilde{\phi}^\dagger L_f$$

- What are the low-energy excitations of ψ? What is the $\mathcal{L}_{EFT}$?

Low-energy modes for Majorana neutrinos

- A Majorana fermion obeys $\psi = \psi^c = C\bar{\psi}^T$

$$\psi = \left(\frac{1+\not{v}}{2}\right)\psi + \left(\frac{1-\not{v}}{2}\right)\psi \equiv \psi_< + \psi_>$$

- Taking the charge conjugate

$$\psi^c = \left(\frac{1-\not{v}}{2}\right)(C\gamma^0\psi^*_<) + \left(\frac{1+\not{v}}{2}\right)(C\gamma^0\psi^*_>)$$

$$\psi_< = C\gamma^0\psi^*_> \,, \quad \psi_> = C\gamma^0\psi^*_<$$

- The field N matches $\psi_<$ in the fundamental theory
- N annihilates a heavy Majoarana fermion or anti-fermion

Low-energy modes for Majorana neutrinos

- What is the propagator for N?

$$\langle 0| T(N^\alpha(x)\bar{N}^\beta(y))|0\rangle = \left(\frac{1+\slashed{v}}{2}\right)^{\alpha\beta} \int \frac{d^4k}{(2\pi)^4}\, e^{-ik(x-y)}\, \frac{i}{v\cdot k + i\epsilon}$$

- The free EFT Lagrangian reads

$$\mathcal{L}^{(0)}_{\mathrm{N}} = \bar{N}\, iv\cdot\partial\, N$$

- In a reference frame where the heavy particle is at rest up to to $k \ll M$

$$\mathcal{L} = N^\dagger i\partial_0 N + \sum_i c_n\left(\frac{\mu}{M}\right)\frac{\mathcal{O}(\mu, T)_n}{M^{d_n-4}} + \mathcal{L}_{light}$$

Observations

- In the heavy particle sector: expansion in 1/M
- Contribution of higher order operators are counted in power of T/M
- The Wilson coefficients may be computed setting T=0, in vacuum

In order to test the EFT approach: $M \gg T \gg M_W$

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}\bar{\psi} i \partial\!\!\!/ \psi - \frac{M}{2}\bar{\psi}\psi - F_f \bar{L}_f \tilde{\phi} P_R \psi - F_f^* \bar{\psi} P_L \tilde{\phi}^\dagger L_f \,,$$

- thermal width at finite T,

$$\Gamma_0 = \frac{|F|^2 M}{8\pi} \rightarrow \Gamma(T) = \Gamma_0 + (\text{thermal corrections})$$

EFT strategy:

1) Matching procedure corresponds to one loop calculation at $T = 0$

   $\Rightarrow$ Effective interactions between N and SM fields

2) Thermal effects encoded in tadpole diagrams at $T \neq 0$

   $\Rightarrow$ Thermal width obtained from $\langle N(x) N(0) \rangle_T$

EFT at work: part I

Matching computation at T=0

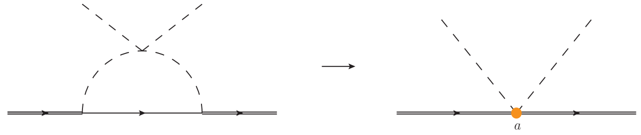
1) The low-energy Lagrangian is organized as follows:

$$\mathcal{L} = N^\dagger \left(i\partial_0 - i\frac{\Gamma_0}{2} \right) N + \frac{\mathcal{L}^{(1)}}{M} + \frac{\mathcal{L}^{(2)}}{M^2} + \frac{\mathcal{L}^{(3)}}{M^3} + \mathcal{O}\left(\frac{1}{M^4}\right)$$

2) By dimensional analysis the thermal corrections scale as

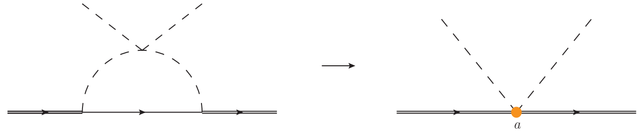
$$\delta\Gamma_{(1)} \propto \frac{T^2}{M}\,, \quad \delta\Gamma_{(2)} \propto \frac{T^3}{M^2}\,, \quad \delta\Gamma_{(3)} \propto \frac{T^4}{M^3}$$

- By symmetry, the leading dimension 5 operator: $\mathcal{L}^{(1)} = aN^\dagger N\phi^\dagger\phi$



EFT at Work II

- the matching is performed in the reference frame $v^\mu = (1, \vec{0}\,)$
- we need the $\mathrm{Im}(a) \Rightarrow$ the imaginary part of $\mathcal{D}$
- incoming and outgoing SM particles carry the same momentum $q^\mu \ll M$

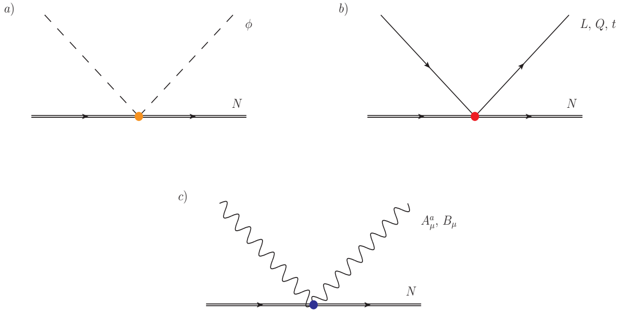


$$\int d^4x\, e^{ip\cdot x} \int d^4y \int d^4z\, e^{iq\cdot(y-z)}\, \langle\Omega| T(\psi^\mu(x)\bar{\psi}^\nu(0)\phi_m(y)\phi_n^\dagger(z))|\Omega\rangle\Big|_{p^\mu=(M+i\epsilon,\vec{0}\,)}$$

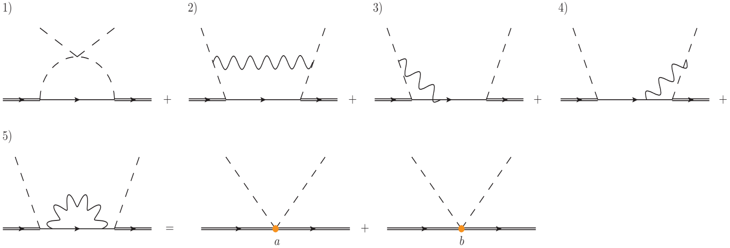
$$M \gg \Lambda \gg T \to 0\,, \quad \mathrm{Im}(a) = -\frac{3}{8\pi}\lambda|F|^2$$

Sub-leading operators

- Higher order operators in the effective Lagrangian
- Higgs, fermions (quarks and leptons) and gauge bosons effective vertices

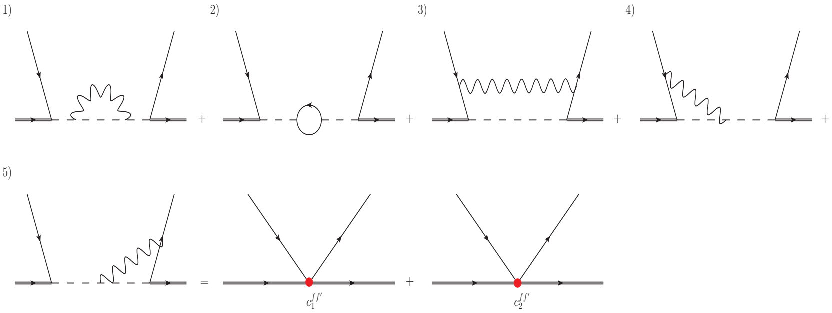


Matching: Higgs

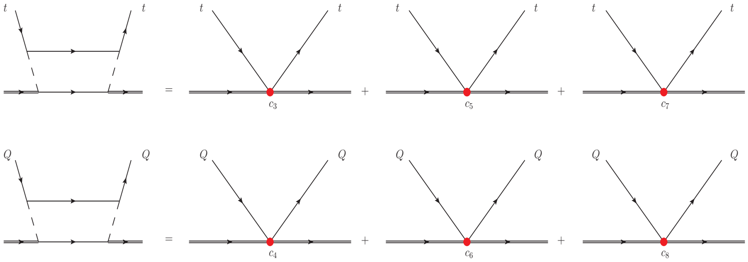


$$\mathcal{O}_\phi^{(1)} = \frac{a}{M} N^\dagger N \phi^\dagger \phi \,, \quad \mathcal{O}_\phi^{(3)} = \frac{b}{M^3}\, \bar{N} N \left(v \cdot D\phi^\dagger\right)\left(v \cdot D\phi\right)$$

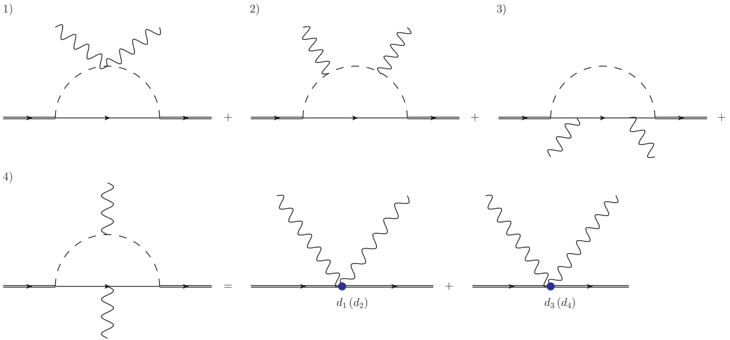
Matching: Leptons



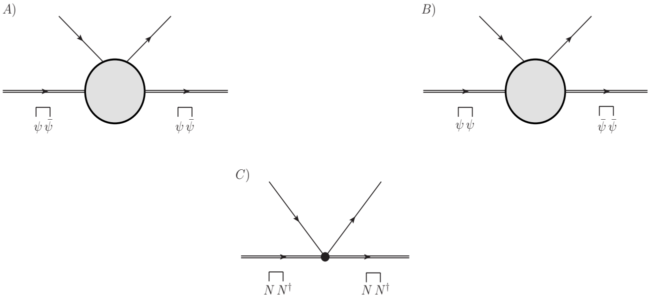
Matching: Quarks



Matching: Gauge bosons



Matching Majorana matrix elements



- Majorana fermions may be contracted in two different ways: A and B

  due to $\langle \psi\psi \rangle$, $\langle \bar{\psi}\bar{\psi} \rangle$

- In the EFT: one combination for the diagrams, $\langle NN \rangle = \langle \bar{N}\bar{N} \rangle = 0$
- Majorana paring enters the matching calculation (not present in HQET)

Thermal width I

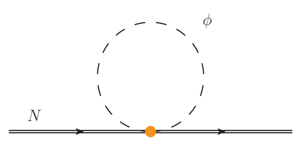
- The thermal width can be computed from

$$\int d^4x\ e^{ik\cdot x}\,\langle T(N^\alpha(x)N^{\dagger\,\beta}(0))\rangle_T^{\rm int}$$

- In the $v^\mu = (1,\vec{0})$ frame, the Majorana neutrino has the form

$$\left(\frac{1+\gamma_0}{2}\right)^{\alpha\beta}\frac{iZ}{k^0-E+i\Gamma/2} = \left(\frac{1+\gamma_0}{2}\right)^{\alpha\beta} Z\left[\frac{i}{k^0+i\epsilon} - \left(iE+\frac{\Gamma}{2}\right)\left(\frac{i}{k^0+i\epsilon}\right)^2 + \cdots\right]$$

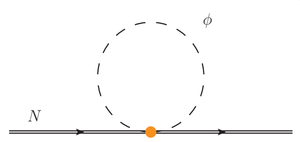
1) E, Γ and Z are specified by self-energy diagrams
2) $Z = 1$ because tadpoles do not depend on incoming momentum



Thermal width II

Thermal width in the EFT

- By expanding the Majorana neutrino propagator in $\mathcal{L}_{\rm EFT}$



$$\Gamma_\phi = 2\frac{{\rm Im}\, a}{M}\langle \phi^\dagger(0)\phi(0)\rangle_T = -\frac{\lambda |F|^2 M}{8\pi}\left(\frac{T}{M}\right)^2$$

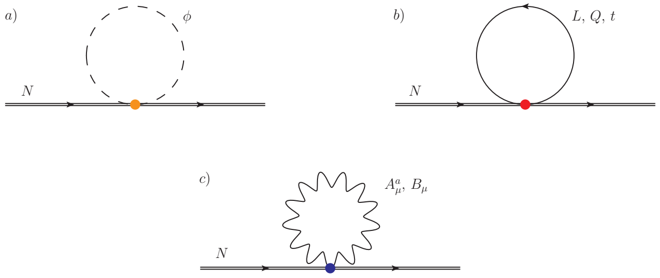
- Higgs propagator in the Real Time Formalism (RTF)

$$i\Delta_{11}(x-y) = \int \frac{d^4k}{(2\pi)^4}\left[\frac{i}{k^2+i\epsilon} + (2\pi)n_B(|k_0|)\delta(k^2)\right] e^{-ik\cdot(x-y)}$$

- Neutrino field of type-2 decouples in the heavy mass limit

$\Rightarrow$ The doubling of the RTF does not enter the computation

- Thermal tadpoles induced by the sub-leading operators



EFT for Majorana fermions works:

$$\Gamma_T = \frac{|F|^2 M}{8\pi}\left\{-\lambda\left(\frac{T}{M}\right)^2 - \frac{\pi^2}{80}\left(\frac{T}{M}\right)^4(3g^2+g'^2) - \frac{7\pi^2}{60}\left(\frac{T}{M}\right)^4|\lambda_t|^2 + \mathcal{O}\left(\frac{T}{M}\right)^6\right\}$$

M. Laine and Y. Schroeder (2012)

Summary

- Non-relativistic regime may be relevant for leptogenesis
- Interactions occur in a thermal medium

$\Downarrow$

EFT for non-relativistic Majorana fermions at finite temperature

- We test the EFT approach by reproducing the neutrino thermal width

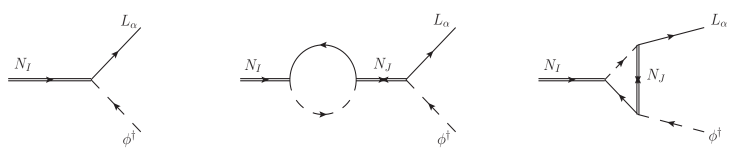
1) one-loop $T = 0$ computation for the matching
2) thermal tadpoles (one-loop)

$\Rightarrow$ Relativistic and thermal correction factorize

CP asymmetry

- Thermal corrections to the CP asymmetry within EFT

$$\epsilon = \sum_{I,f} \frac{\Gamma(N_I \to \ell_f + X) - \Gamma(N_I \to \bar{\ell}_f + X)}{\Gamma(N_I \to \ell_f + X) + \Gamma(N_I \to \bar{\ell}_f + X)}$$



1) We consider two Majorana neutrinos: N_1 and N_2
2) We focus on the direct CP asymmetry (vertex diagram)

CP asymmetry and medium effects

- Thermal effects for the CP asymmetry have been already considered

  M. Garny, A. Hohenegger, A. Kartavtsev and M. Lindner (2009)

  M. Garny, A. Hohenegger and A. Kartavtsev (2010)

  B. Garbrecht, F. Gautier and J. Klaric (2014)

Correction form EFT

- Exponentially suppressed corrections are neglected form the beginning
- We can instead obtain corrections like

$$g_{\rm SM} \times \left(\frac{T}{M}\right)^n$$

EFT strategy

Fundamental Lagrangian

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}\bar{\psi}_I i\partial\!\!\!/ \psi_I - \frac{M_I}{2}\bar{\psi}_I\psi_I - F_{fI}\bar{L}_f\tilde{\phi}P_R\psi_I - F^*_{If}\bar{\psi}_I P_L\tilde{\phi}^\dagger L_f$$

EFT Lagrangian for N_1 and N_2: $M \gg T \gg M_W, \Delta$

$$\mathcal{L}_{\rm EFT} = \mathcal{L}_{\rm SM} + \bar{N}_I\left(iv\cdot\partial - \delta M_I\right)N_I - \frac{i\Gamma^{T=0}_{IJ}}{2}\bar{N}_I N_J + \frac{a_{IJ}}{M}\bar{N}_I N_J\phi^\dagger\phi + \ldots$$

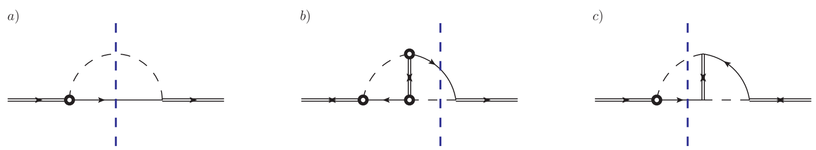
$$\delta M_1 = 0\,,\ \delta M_2 = \Delta$$

- Almost degenerate neutrino masses
- Direct ϵ is related to to $\Gamma^{T=0}_{II}$ and a_{II} (and possible corrections to them)

1) Matching calculation to fix the Wilson coefficients, $T=0$
2) Thermal effects encoded in thermal tadpoles

Asymmetry at $T = 0$

- Diagrams relevant for the direct asymmetry



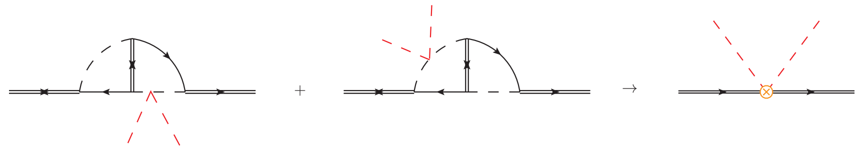
- The CP asymmetry can be written as

$$\epsilon_1^d = 2\,\frac{\mathrm{Im}(B)\mathrm{Im}\left[(F_1F_2^*)^2\right]}{|F_1|^2} = \frac{(1-2\ln 2)\mathrm{Im}\left[(F_1F_2^*)^2\right]}{8\pi|F_1|^2}$$

- By using techniques at $T = 0$ one can extract $\mathrm{Im}(B)$

First step: Matching

- We consider dimension 5 operators: interaction with Higgs from the medium
- We match matrix element in the fundamental end low-energy theory

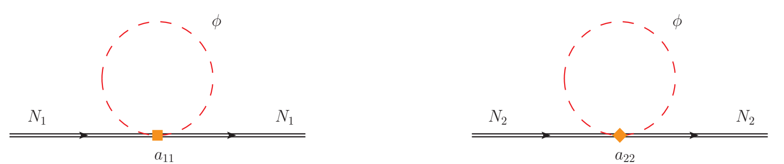


- First order SM coupling: $-i\mathcal{D} \propto \lambda |F|^4$

- The scale M is integrated out: $M \gg \Lambda \gg T \to 0$
- The cuts put on-shell either leptons or anti-leptons
- Cutting rules at zero temperature

Second step: thermal tadpoles

- The Higgs tadpole induces the thermal corrections



The asymmetry in N_1 decays is generated from:

$$\Gamma(N_1 \to \ell + X) - \Gamma(N_1 \to \bar{\ell} + X) = 2 \left(\frac{\mathrm{Im}\, a_{11}^{\ell}}{M} - \frac{\mathrm{Im}\, a_{11}^{\bar{\ell}}}{M} \right) \langle \phi^{\dagger}(0)\phi(0) \rangle_T$$

- Factorization of relativistic and thermal corrections

Final result

Final result for the CP asymmetry

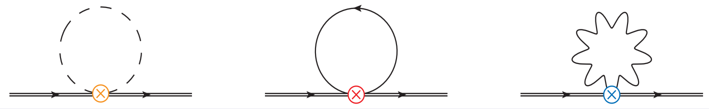
- By considering both the decays of N_1 and N_2 we obtain

$$\epsilon_T^d = -\frac{\mathrm{Im}\left[(F_1 F_2^*)^2\right]}{16\pi}\frac{|F_2|^2 - |F_1|^2}{|F_1|^2|F_2|^2}\left[\lambda(1-2\ln 2) + (3g^2 + g'^2)\frac{2-\ln 2}{24}\right]\left(\frac{T}{M}\right)^2$$

- Dimensional analysis in the EFT predicts the counting $\left(\frac{T}{M}\right)^2$

$$\langle\phi^\dagger(0)\phi(0)\rangle_T \sim T^2$$

- The sub-leading contributions, $\mathcal{O}\left(\frac{T}{M}\right)^4$ are related to

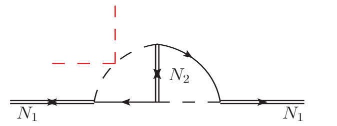


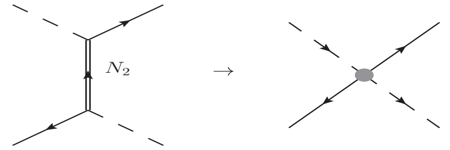
$\rightarrow$ thermal condensates scale as T^4

Hierarchical case for N_1 and N_2

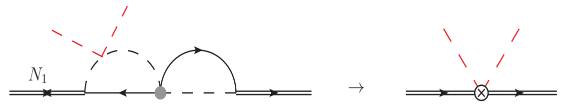
- $M_1 \simeq M_2$: the direct CP asymmetry may be not the main contribution
- $M_1 \ll M_2$: the vertex and wave function diagrams are equally important

- We can use the EFT approach
  1) Integrate out the bigger neutrino mass M_2





  2) Integrate out the smaller neutrino mass M_1



Conclusions

- Non-relativistic regime may be relevant for leptogenesis
- Interactions occur in medium

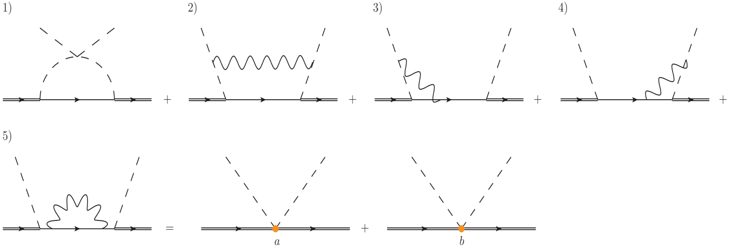
- Formulation of an EFT to deal with Majorana neutrinos in medium
- Test the EFT: neutrino thermal width up to $\mathcal{O}\left(\frac{T}{M}\right)^4$

  2-loops ($T \neq 0$) $\rightarrow$ 1-loop (T=0) + 1-loop (thermal tadpoles)

- Address the CP asymmetry in the EFT formalism
- Matching calculation (cutting rules at $T = 0$) and thermal tadpoles ($T \neq 0$)
- Calculation of thermal corrections to ϵ due to the vertex diagram

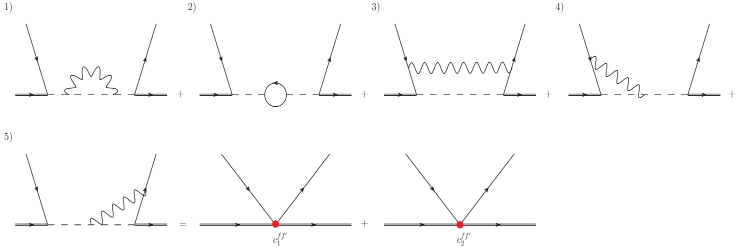
$$\epsilon(T) = \epsilon_0 \left[1 + g_{\rm SM} \times \left(\frac{T}{M}\right)^n\right]$$

Matching: Higgs



$$\mathcal{O}_{\phi}^{(1)} = \frac{a}{M} N^{\dagger} N \phi^{\dagger} \phi \,, \quad \mathcal{O}_{\phi}^{(3)} = \frac{b}{M^3} \, \bar{N} N \left(v \cdot D\phi^{\dagger}\right) \left(v \cdot D\phi\right)$$

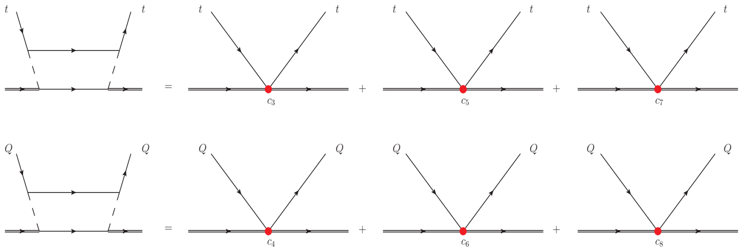
Matching: Leptons



$$\mathcal{O}_L^{(3)} = c_1^{ff'} \left[\left(\bar{N} P_L \, iv \cdot D L_f \right) \left(\bar{L}_{f'} P_R N \right) + \left(\bar{N} P_R \, iv \cdot D L^c_{f'} \right) \left(\bar{L}^c_f P_L N \right) \right]$$

$$+ c_2^{ff'} \left[\left(\bar{N} P_L \, \gamma_\mu \gamma_\nu \, iv \cdot D L_f \right) \left(\bar{L}_{f'} \, \gamma^\nu \gamma^\mu \, P_R N \right) \left(\bar{N} P_R \, \gamma_\mu \gamma_\nu \, iv \cdot D L^c_{f'} \right) \left(\bar{L}^c_f \, \gamma^\nu \gamma^\mu \, P_L N \right) \right]$$

Matching: Quarks

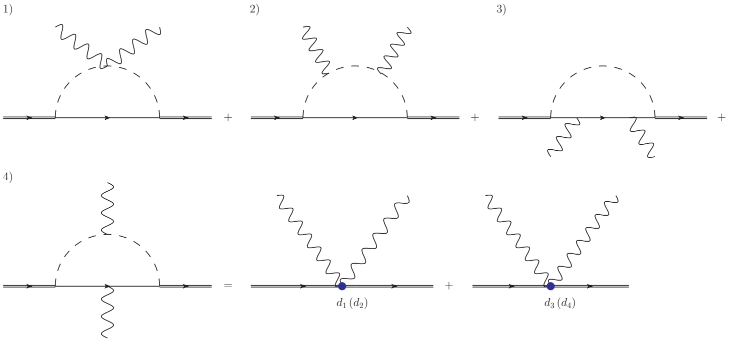


$$\mathcal{O}^{(3)}_{Q,t} = c_3\, \bar{N}N\, \left(\bar{t}P_L\, v^\mu v^\nu \gamma_\mu\, iD_\nu t\right) + c_4\, \bar{N}N\, \left(\bar{Q}P_R\, v^\mu v^\nu \gamma_\mu\, iD_\nu Q\right)$$

$$+ c_5\, \bar{N}\gamma^5\gamma^\mu\, N\, \left(\bar{t}P_L\, v\cdot\gamma\, iD_\mu t\right) + c_6\, \bar{N}\gamma^5\gamma^\mu\, N\, \left(\bar{Q}P_R\, v\cdot\gamma\, iD_\mu Q\right)$$

$$+ c_7\, \bar{N}\gamma^5\gamma^\mu\, N\, \left(\bar{t}P_L\, \gamma_\mu\, iv\cdot Dt\right) + c_8\, \bar{N}\gamma^5\gamma^\mu\, N\, \left(\bar{Q}P_R\, \gamma_\mu\, iv\cdot DQ\right)$$

Matching: Gauge bosons



$$\mathcal{O}^{(3)}_{A^\mu,B^\mu} = -d_1\ \bar{N}N\,v^\mu v_\nu W^a_{\alpha\mu}W^{a\,\alpha\nu}\ -d_2\ \bar{N}N\,v^\mu v_\nu F_{\alpha\mu}F^{\alpha\nu}$$

$$+d_3\ \bar{N}N\,W^a_{\mu\nu}W^{a\,\mu\nu}\ +d_4\ \bar{N}N\,F_{\mu\nu}F^{\mu\nu}$$

Momentum dependent operator

- If the Majorana neutrino is not at rest $\Rightarrow$ momentum dependent operator

$$\mathcal{O}^{(3)}_{\vec{k}} = -\frac{1}{2M^3} a\, \bar{N}\left[\partial^2 - (v\cdot\partial)^2\right] N\, \phi^\dagger\phi$$

- The Wilson coefficient is fixed by the dispersion relation

$$\bar{N}N\left(\sqrt{(M+\delta m)^2 + \vec{k}^{\,2}} - M\right) = \bar{N}N\left(\delta m + \frac{\vec{k}^{\,2}}{2M} - \delta m\,\frac{\vec{k}^{\,2}}{2M^2} + \dots\right)$$

where $\delta m = -a\frac{\phi^\dagger\phi}{M}$

Thermal tadpole contains the Higgs condensate

$$\Gamma_{\vec{k}} = 2\frac{\mathrm{Im}\, a}{M}\left(-\frac{\vec{k}^{\,2}}{2M^2}\right)\langle\phi^\dagger(0)\phi(0)\rangle_T = -\frac{\mathrm{Im}\, a}{6}\,\frac{\vec{k}^{\,2}T^2}{M^3}$$