The Jet Quenching Parameter and Effective Theories

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Outline

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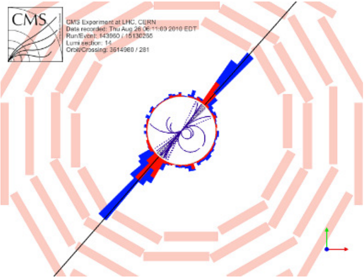
1. Introduction
   - Jets
   - The jet quenching parameter
2. The effective field theory approach
3. An effective theory for the jet
   - Soft-Collinear Effective Theory
   - The Glauber mode
   - Gauge invariance
4. Effective theories for the medium
   - Electrostatic QCD
   - Perturbative calculations
   - Non-perturbative contributions
5. Conclusions

What is what?

- What is jet quenching?
  → Modification of jet observables due to presence of a thermal medium (e.g. quark-gluon plasma)

- What is a jet?
  → A *narrow* cone of hadrons with a *large* energy and a small invariant mass (light hadrons)

  In vacuum:

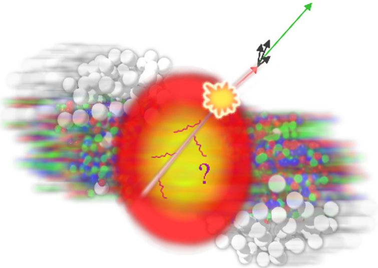


CMS collaboration

- What is the quark-gluon plasma (QGP)?
  → A phase of the strongly interacting matter

Jets in the Quark-Gluon Plasma

- Jets have a clear experimental signature
- They are produced by hard interactions before the formation of the plasma
  $\rightarrow$ Production calculable at $T = 0$



lbl.gov

- Subsequently propagate through the plasma
  $\rightarrow$ By comparison with jets in p-p collisions the properties of the QGP can be analyzed

Plasma Effects on the Jet

Two types of interaction

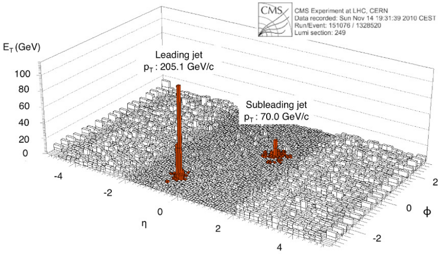
- **Radiative energy loss** through medium induced gluon radiation (radiated gluons are again subject to in medium interactions)
- **Jet broadening** without energy loss, i.e. change of momentum perpendicular to initial jet direction through interaction with medium constituents

  $\rightarrow$ Both effects interfere and are relevant to the so-called **jet quenching**

- For two jets in p-p collisions one expects them to be back-to-back
- In heavy ion collisions on the other hand, one of the jets can be significantly suppressed due to interactions with the QGP

Experimental Results

- Jet quenching has been observed at PHENIX, STAR (RHIC) and ATLAS, CMS (LHC)



CMS collaboration

- Measurable quantity of interest: Nuclear modification factor

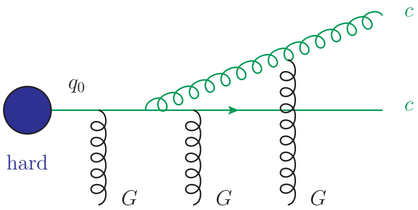
$$R_{AA} = \frac{d\sigma_{AA}(p_T, y)/dp_T dy}{\langle \sigma_{NN} T_{AA} \rangle d\sigma_{pp}(p_T, y)/dp_T dy}$$

Ratio of observed hadrons in heavy ion collisions to p-p collisions normalized to number of nucleon binary collisions

Theoretical Considerations

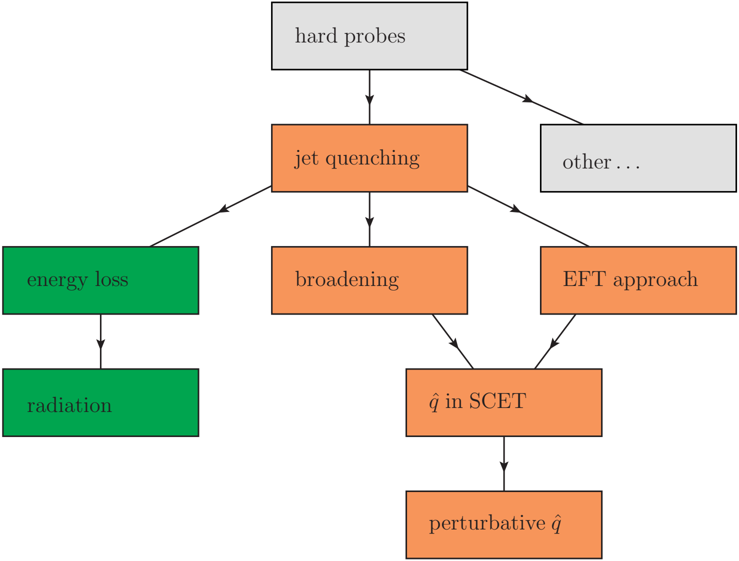
- There are several approaches to calculate the effect of the medium on jets due to

  Baier, Dokshitzer, Peigne, Schiff, Zakharov, Armesto, Salgado, Wiedemann, Gyulassy, Levai, Vitev, Guo, Wang, Arnold, Moore, Yaffe, . . .



- How to characterize the medium?

Schematic


hard probes
jet quenching
other . . .
energy loss
broadening
EFT approach
radiation
$\hat{q}$ in SCET
perturbative $\hat{q}$

The Jet Quenching Parameter

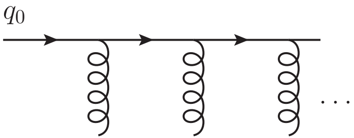
- One way to parameterize effect of the medium is to introduce a **jet quenching parameter**
- It corresponds to the change of the momentum perpendicular to the original direction of the jet parton per distance traveled

  When describing the broadening of the $k_\perp$-distribution while travelling a distance through the medium by a diffusion equation, $\hat{q}$ is related to the diffusion constant
- Introduce $P(k_\perp)$, the probability to acquire a perpendicular momentum $k_\perp$ after travelling through a medium with length L

$$P(k_\perp) \sim \frac{1}{\hat{q}L} e^{-\frac{k_\perp^2}{\hat{q}L}}$$

The Jet Quenching Parameter

- We will find that the Fourier transform $P(x_\perp)$ exponentiates
$$P(x_\perp) \sim e^{C(x_\perp)L}$$
where $C(x_\perp)$ is the collision kernel



- The jet quenching parameter may then be defined as
$$\hat{q} = \int \frac{d^2k_\perp}{(2\pi)^2}\, k_\perp^2\; C(k_\perp)$$
where the range of integration is restricted by a process-dependent cut-off

Scope

- Does **not** include collinear radiation which changes the energy of the parton significantly
- Assume that the final virtuality is determined through **medium interactions** and not the initial hard process
- Assume a **thermalized** medium

Goals

Find field theoretic definition of $\hat{q}$ using an effective field theory approach

Systematic calculation of the contributions to $\hat{q}$ in the weak coupling regime

Outline

The relevant scales

- Several scales appear in the process, most notably

  The energy of the jet Q

  The scale of the medium (temperature) T

  Thermal scales, such as the Debye mass $m_D \sim g T$

  the chromomagnetic mass $g_E \sim g^2 T$ (magnetostatic screening)
- In the weak coupling limit (g small) these scales are ordered by their size

Approach

Introduce a series **effective field theories** to

- transparently derive factorization
- obtain a systematic expansion in terms of ratios of scales
- resum possible large logarithms of ratios of scales

The effective field theory approach

- The appropriate field theories are
- full (perturbative) QCD for hard interactions at the scale Q (creation of the primary jet particle)
- **Soft-Collinear Effective Theory** (SCET) for the description of a jet interacting with soft particles at the scale T

  Bauer et al. '01; Beneke at al. '02
- **Electrostatic QCD** (EQCD) for interactions in a thermalized medium where the scale T has been integrated out

  Braaten '95
- **Magnetostatic QCD** (MQCD) for interactions at the non-perturbative scale $g^2 T$

  Braaten '95

Outline

Soft-Collinear Effective Theory

- Small dimensionless ratio $\lambda = T/Q \ll 1$
- Classify modes by the scaling of their momentum components in the different light-cone directions $(n, \bar{n})$

  $(p^+, p^-, p_\perp) = (Q, Q, Q) \sim (1, 1, 1)$ is called hard

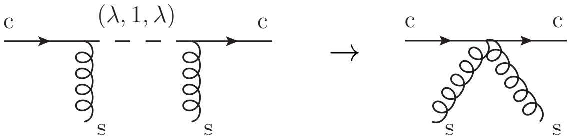
  $(p^+, p^-, p_\perp) = (T, T, T) \sim (\lambda, \lambda, \lambda)$ is called soft

  $(p^+, p^-, p_\perp) \sim (\lambda^2, 1, \lambda)$ is called collinear
- **Jets** have a collinear momentum, i.e., they have a large momentum component in one light cone direction, but only a small invariant mass
- Integrate out the hard modes and the off-cone components of the collinear modes to find the **SCET Lagrangian** for collinear fields

$$\mathcal{L} = \bar{\xi} i \bar{n} \cdot D \frac{\not{n}}{2} \xi + \bar{\xi} i \not{D}_\perp \frac{1}{in \cdot D} i \not{D}_\perp \frac{\not{n}}{2} \xi + \mathcal{L}_{\text{Y.M.}}, \quad iD = i\partial + gA$$

SCET Modes

- Soft: Typical representative of the medium; no leading power collinear-soft interaction in the SCET Lagrangian, but



(only if $+$ components of soft momenta add up to λ^2)

- Other possible modes interacting with a collinear quark $(p^+, p^-, p_\perp) \sim (\lambda^2, \lambda^2, \lambda^2)$ is called ultrasoft

  Decouple at leading power as proven in Bauer et al. '01

In-medium Interactions

- The most relevant mode for jet broadening

  $(p^+, p^-, p_\perp) \sim (\lambda^2, \lambda^2, \lambda)$ is called **Glauber**

  Necessary for consistence in exclusive Drell-Yan with spectator interactions Bauer, Lange, Ovanesyan '10

  and also for interactions with a medium Idilbi, Majumder '08

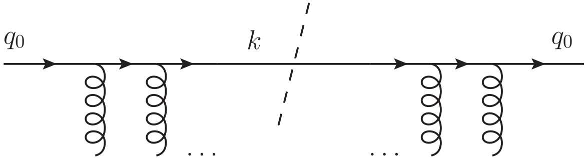
  "longitudinal" **Glauber** $(\lambda^2, \lambda, \lambda)$: Also found to be important Ovanesyan, Vitev '11

  recently in the context of longitudinal drag Qin, Majumder '12
- Introduce Glauber field into the SCET Lagrangian as an effective classical field of the medium particles

Calculation of $P(k_\perp)$

- Determine the probability $P(k_\perp)$ by calculating the amplitude for the interaction of the collinear quark with gluons from the medium
- First attempt: Use SCET$_G$ in **covariant gauge**

  D'Eramo, Liu, Rajagopal '10
- Use optical theorem to determine scattering amplitude



- Initially on-shell quark scattering on an arbitrary number of medium particles via Glauber exchange
- Type of source relevant for eikonalization

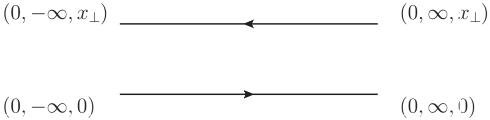
$P(k_\perp)$ in covariant gauge

- Result is the Fourier transform of the medium averaged expectation value of two Wilson lines

$$P(k_\perp) = \int d^2 x_\perp e^{ik_\perp \cdot x_\perp} \frac{1}{N_c} \left\langle \mathrm{Tr} \left[W_F^\dagger[0, x_\perp] W_F[0,0] \right] \right\rangle$$

$$W_F[y^+, y_\perp] = \mathcal{P} \left\{ \exp \left[ig \int_{-\infty}^{\infty} dy^- A^+(y^+, y^-, y_\perp) \right] \right\}$$

- Agrees with known results Casalderrey-Solana, Salgado '07 but pay attention to operator ordering



- Not gauge invariant ($W_F = 1$ in light-cone gauge $A^+ = 0$)

Changes in arbitrary gauge

Goal

Want to show that SCET_G is complete and find a gauge invariant expression of $P(k_\perp)$ for further calculations (e.g. lattice)

- In singular gauges, such as light-cone gauge the scaling of the Glauber field is **different**

  Idilbi, Majumder '08; Ovanesyan, Vitev '11

  $A_\perp^{\text{cov}} \ll A_\perp^{\text{lcg}}$
- This can be traced back to the factor $k_\perp/[k^+]$ appearing in the Fourier transform of the gluon propagator in light-cone gauge (the square brackets indicate an appropriate regularization for the light-cone singularity)
- Additional leading power interaction term in the Lagrangian becomes relevant

$$\bar{\xi} i\not{D}_\perp \frac{1}{Q} i\not{D}_\perp \frac{\not{\bar{n}}}{2} \xi$$

Changes in arbitrary gauge

- Additional vertices for collinear-Glauber interaction

- Summing over any number of gluon interactions, we find in light-cone gauge

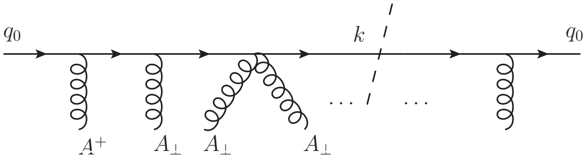
$$
\begin{aligned}
P(k_\perp) \;=\; & \frac{1}{N_c}\int d^2x_\perp\, e^{ik_\perp\cdot x_\perp} \\
& \left\langle \mathrm{Tr}\left[T^\dagger(0,-\infty,x_\perp)\, T(0,\infty,x_\perp)\; T^\dagger(0,\infty,0)\, T(0,-\infty,0)\right]\right\rangle
\end{aligned}
$$

- with
  $T(x_+,\pm\infty,x_\perp) = \mathcal{P}\, e^{-ig\int_{-\infty}^{0} ds\; l_\perp\cdot A_\perp(x_+,\pm\infty,x_\perp+l_\perp s)}$
  the transverse Wilson line

Results

- Wilson lines in the perpendicular plane at $\pm\infty^-$ for light-cone gauge

Results combined

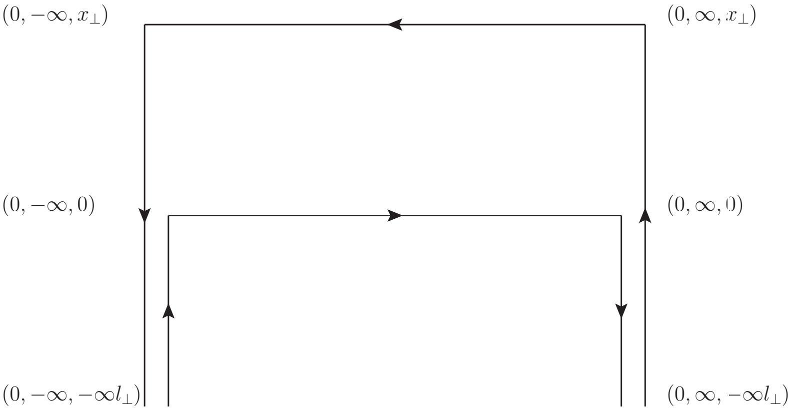


- Combining the results with the ones in covariant gauge we find

$$
\begin{aligned}
P(k_\perp) \;\; = \;\; & \frac{1}{N_c} \int d^2x_\perp \, e^{ik_\perp \cdot x_\perp} \\
& \left\langle \mathrm{Tr} \left[T^\dagger(0, -\infty, x_\perp) \, W_F^\dagger[0, x_\perp] \, T(0, \infty, x_\perp) \right.\right. \\
& \left.\left. T^\dagger(0, \infty, 0) \, W_F[0, 0] \, T(0, -\infty, 0) \right] \right\rangle
\end{aligned}
$$

- Note that for certain regularizations of the light-cone singularity of the gluon propagator, the Glauber field might vanish at either $+\infty^-$ or $-\infty^-$ even in light-cone gauge
  Liang, Wang, Zhou '08

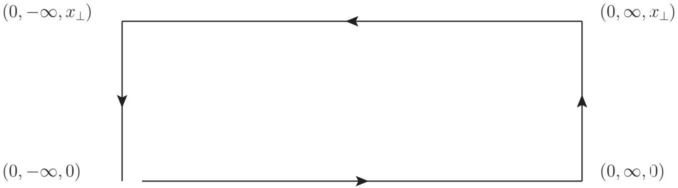
Results combined



Mi.B., Brambilla, Escobedo, Vairo '12

Results combined

- Transverse Wilson lines combine to



- Fields on the lower line are time ordered, the ones on the upper line anti-time ordered
  $\rightarrow$ Use **Schwinger-Keldysh contour** in path integral formalism

Next steps

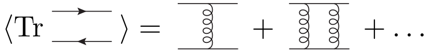
- Consider emission of collinear gluons in EFT framework.
- Actually calculate $\hat{q}$ in the weak coupling regime.

Outline

Thermal Field Theory

- Probability $P(x_\perp)$ is related to **thermal expectation value** of a light-cone Wilson loop
- Assume a thermalized medium and a weak coupling g and calculate $\hat{q}$ in thermal field theory
- Since the Wilson loop extends along the light-cone, in principle one needs analytic continuation in the **imaginary time formalism** or a doubling of degrees of freedom in the **real time formalism**

  $\rightarrow$ Calculate $P(x_\perp)$ in perturbation theory



- Use covariant gauge

Effective Thermal Field Theory

- Due to the appearance of additional thermal scales $m_D \sim gT$ and $g_E \sim g^2 T$ (electro- and magnetostatic screening) large logarithms may appear, invalidating the perturbative expansion
  $\rightarrow$ Introduce a set of effective field theories to separate the scales
  Braaten '95
- In the imaginary time formalism an equal-time propagator can be written

$$G(t=0,\vec{x}) = T\sum_n \int \frac{d^3p}{(2\pi)^3}\, e^{i\vec{p}\cdot\vec{x}} G_E(\omega_n,\vec{p})$$

$$\omega_n = 2\pi n T \qquad \text{Matsubara frequency}$$

$$G_E(\omega_n,\vec{p}) \sim \frac{1}{\omega_n^2+\vec{p}^2} \qquad \text{Euclidean propagator}$$

- Integrate out the Matsubara modes $n>0$ ($\sim \pi T$) and redefine $A^\mu \rightarrow \sqrt{T}A^\mu \rightarrow$ **electrostatic QCD** (EQCD)

Electrostatic QCD

- EQCD is 3D Euclidean Yang-Mills coupled to massive scalar A^0 with mass $m_E \sim gT$ and coupling $g_E \sim g^2 T$
- EQCD Lagrangian

$$\mathcal{L}_{\mathrm{EQCD}} = \frac{1}{4} G_{ij}^a G_{ij}^a + \frac{1}{2}(D_i A_0)^2 + \frac{1}{2} m_E^2 A_0^2$$

- Propagators in EQCD
  $G^{00} = -1/(\vec{q}^2 + m_E^2)$ and $G^{ij} = \delta^{ij}/\vec{q}^2$
- In principle applicable for calculating $P(k_\perp)$ if $k_\perp \sim gT$
- In order to apply EQCD to light-like correlators, **deform contour** to be slightly space-like ($v = 1 + \epsilon$), then boost to equal time and use imaginary time formalism Caron-Huot '08
- Soft modes are not sensitive to the precise velocity of the jet parton

Perturbative Contributions to $\hat{q}$

- Known perturbative contributions to $\hat{q}$
- Interference of loop and power expansion

  LO $\sim g^4 T^3$ Arnold, Xiao '08 (from $k_\perp \sim T$ and $k_\perp \sim gT$)

  NLO $\sim g^5 T^3$ Caron-Huot '08 (from $k_\perp \sim gT$ with loops)
- The region for $k_\perp \sim gT$ can be derived from the Wilson loop in EQCD (extending along the z direction and containing the fields $A_E^+(x_\perp, z) = (A_E^0 + A_E^3)/\sqrt{2}$)

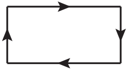
$$\begin{aligned}\hat{q}(q_{\mathrm{max}}) &= \frac{2g^4T^3}{3\pi}\left[\frac{3}{2}\log\left(\frac{T}{m_D}\right) + \frac{7\zeta(3)}{4\zeta(2)}\log\left(\frac{q_{\mathrm{max}}}{T}\right) - 0.105283\right] \\ &+ \frac{g^4T^3}{8\pi^2}\frac{m_D}{T}(3\pi^2 + 10 - 4\log(2))\end{aligned}$$

- At order $g^6 T^3$ non-perturbative contributions from the region $k_\perp \sim g^2 T$ start to appear

Magnetostatic QCD

- So far we have argued that $P(k_\perp)$ for $k_\perp \sim gT$ is related to the expectation value of a Wilson loop 3D-Yang-Mills theory (plus the A^0 field)
- This object is very similar to the expectation value of the 3D static Wilson loop in quarkonium physics
- The probability $P(k_\perp)$ can therefore be related to the **static energy in 3D Yang-Mills theory**

$$P(k_\perp)_{\mathrm{F.T.}} \sim \frac{1}{N_c}\langle \mathrm{Tr}$$



$$\rangle = e^{-h_s(x_\perp)L} \qquad \text{for } L \to \infty$$

- When also integrating out the scale $m_E \sim gT$ (i.e. the field A^0) one arrives at magnetostatic QCD (MQCD) Braaten '95
- The probability $P(k_\perp)$ for $k_\perp \sim g^2 T$ is then exactly related to the static potential Laine '12

Non-perturbative Contributions at NNLO

- In MQCD the jet quenching parameter may be written as

$$\hat{q}|_{g^2T} = -\int \frac{d^2k_\perp}{(2\pi)^2}\, k_\perp^2\, V(k_\perp)$$

  Laine '12

- The static potential V has been calculated on the lattice Luescher, Weisz '02
- Possible to derive the contribution from the non-perturbative region $k_\perp \sim g^2T$ to $\hat{q}$

  Laine '12; Mi.B., Brambilla, Escobedo, Vairo '12
- It is found that the non-perturbative contribution is than the perturbative contribution at NLO

Soft Logarithms

- The relevance of the $g^2 T$ contributions at NNLO manifests in IR divergencies appearing in the EQCD calculation
- The situation is exactly analogous to the static potential in 3D QCD

  Schroeder '99; Pineda, Stahlhofen '10

  There it was demonstrated, that a matching onto a low energy effective theory (pNRQCD) regulates this divergence

$$h_s(x_\perp) = V_s(x_\perp, \mu) + \delta h_s(x_\perp, \mu)$$

- We will use the analogy to the jet quenching case to determine the **logarithmic contributions** at NNLO to $P(k_\perp)$

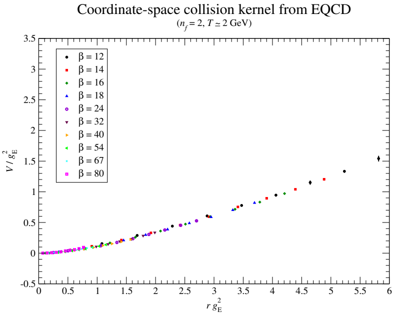
  The role of pNRQCD will be played by MQCD

  $\rightarrow$ work in progress

EQCD on the lattice

- Recently, the full EQCD Wilson loop has been calculated on the lattice

  Panero, Rummukainen, Schaefer '13



- Numerical result: $\hat{q}|_{gT} \approx 0.45(5) g_E^6$ for $T \approx 2\,\mathrm{GeV}$

Outline

Conclusions

- Jet quenching is an useful phenomenon that yields insights into the properties of a quark-gluon plasma
- Due to the appearance of several different scales in practical computations, a series of effective field theories is introduced
- **SCET**$_G$ is a suitable theory to give a gauge invariant field theoretical definition of the jet quenching parameter
- The jet quenching parameter $\hat{q}$ can be expressed as the medium average of two longitudinal and four transverse Wilson lines
- In the appropriate momentum region this medium average can be computed in **EQCD**
- There is an analogy to the static Wilson loop in 3D Yang-Mills
- Perturbative results are available at LO and NLO
- At NNLO the non-perturbative contribution can be determined from the lattice, the logarithms between the two regions derived from the static Wilson loop

Thank you for your attention!

Bonus Slides

Scaling of the Glauber field

- Consider the form of the effective Glauber field

$$A^\mu(x) = \int d^4y \; D_G^{\mu\nu}(x-y)\, f_\nu(y)$$

$$D^{\mu\nu}(x-y) = \int \frac{d^4k}{(2\pi)^4} \, \frac{-i}{k^2 + i\epsilon} \left(g^{\mu\nu} - \frac{k^\mu \bar{n}^\nu + k^\nu \bar{n}^\mu}{[k^+]} \right) e^{-ik(x-y)}$$

- Source f_ν only knows about the soft scale $\sim \lambda^3$

The gauge field at light-cone infinity

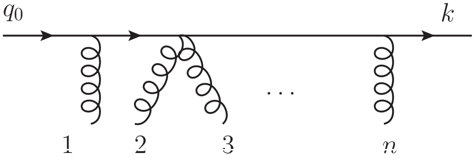
- The gluon field may be decomposed
  $A^i_\perp(x^+, x^-, x_\perp) =$
  $A^{\mathrm{cov},i}_\perp(x^+, x^-, x_\perp) + \theta(x^-) A^i_\perp(x^+, \infty, x_\perp) + \theta(-x^-) A^i_\perp(x^+, -\infty, x_\perp)$
  where $A^{\mathrm{cov},i}_\perp$ corresponds to the non-singular part of the propagator and vanishes at $\pm\infty^-$ and where the leading power comes from the terms at ∞^-

  Echevarria, Idilbi, Scimemi '11

- For $x^- \to \infty$ the field strength must vanish
  $\to A_\perp(x^+, \infty, x_\perp)$ is a pure gauge
  $A_\perp(x^+, \infty, x_\perp) = \nabla_\perp \phi(x^+, \infty, x_\perp)$
  $\phi(x^+, \infty, x_\perp) = -\int_{-\infty}^0 ds\, l_\perp \cdot A_\perp(x^+, \infty, x_\perp + l_\perp s)$
  Belitsky, Ji, Yuan '02

Calculation

- Define the (amputated) diagram with n gluon interactions

$G_n(k) =$ 

- We can calculate this in a recursive fashion

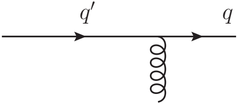
Calculation

- Decompose into fields at $\pm\infty$

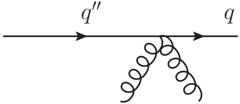
  $G_n(k^-, k_\perp) = \sum_{j=0}^{n} \int \frac{d^4q}{(2\pi)^4}\, G^+_{n-j}(k^-, k_\perp, q)\, \frac{iQ\,\bar{n}\!\!\!/}{2Qq^+ - q_\perp^2 + i\epsilon}\, G^-_j(q)$

  where $G^\pm$ contains only the gluon at $\pm\infty$
- The recursive definition of G^- is then

$$G^-_n(q) = \quad \int \frac{d^4q'}{(2\pi)^4}\, G^-_{n-1}(q')$$

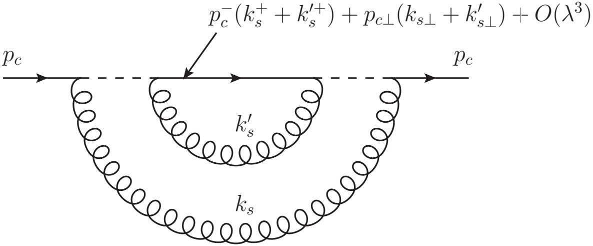


$$+ \quad \int \frac{d^4q''}{(2\pi)^4}\, G^-_{n-2}(q'')$$



- and G^+_n correspondingly

Collinear-soft interactions



- SCET operator

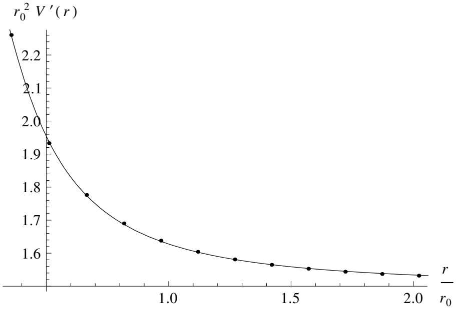
$$\bar{\xi}\not{n} g A_s^+ S \xi$$

- S soft Wilson line

On the lattice

- $\hat{q}$ in terms of the static potential

$$\hat{q}|_{g^2 T} = -(q^*)^2 \int_0^\infty d\lambda\, \lambda^3\, J_0(\lambda) \int_\lambda^\infty \frac{dz}{z^3}\, V\left(\frac{z}{q^*}\right)$$



Luescher, Weisz '02

$$V(r) = \frac{1}{r_0}\left(a\,\frac{r}{r_0} - b\,\frac{r_0}{r} + \dots\right)$$