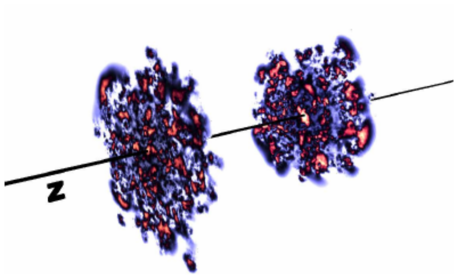
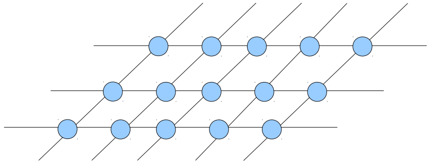
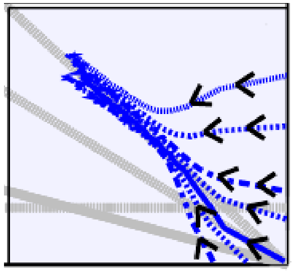
Thermalization process on the lattice







Soeren Schlichting

In collaboration with
J. Berges, K. Boguslavski and R. Venugopalan

MITP Workshop, Mainz 07/29/14

BROOKHAVEN
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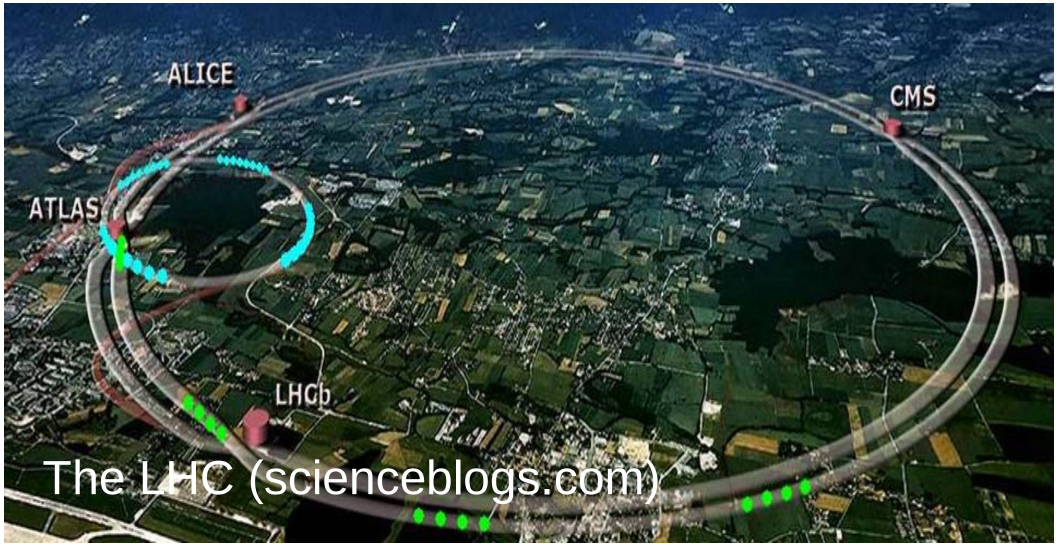
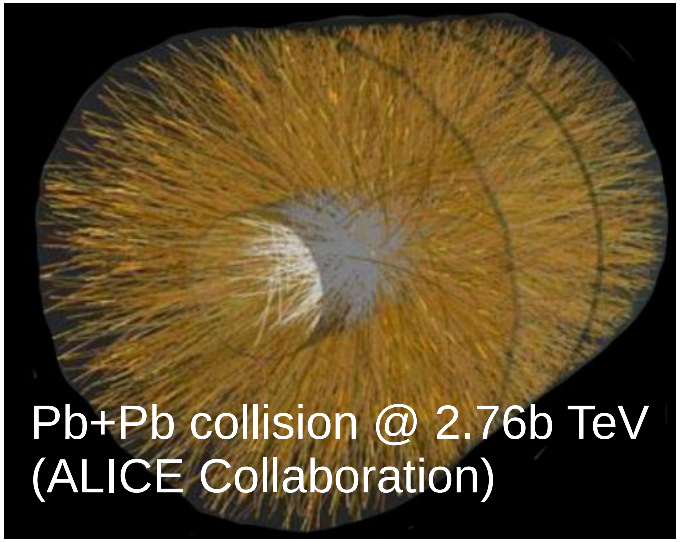
Motivation

Relativistic heavy-ion collision experiments at RHIC and LHC

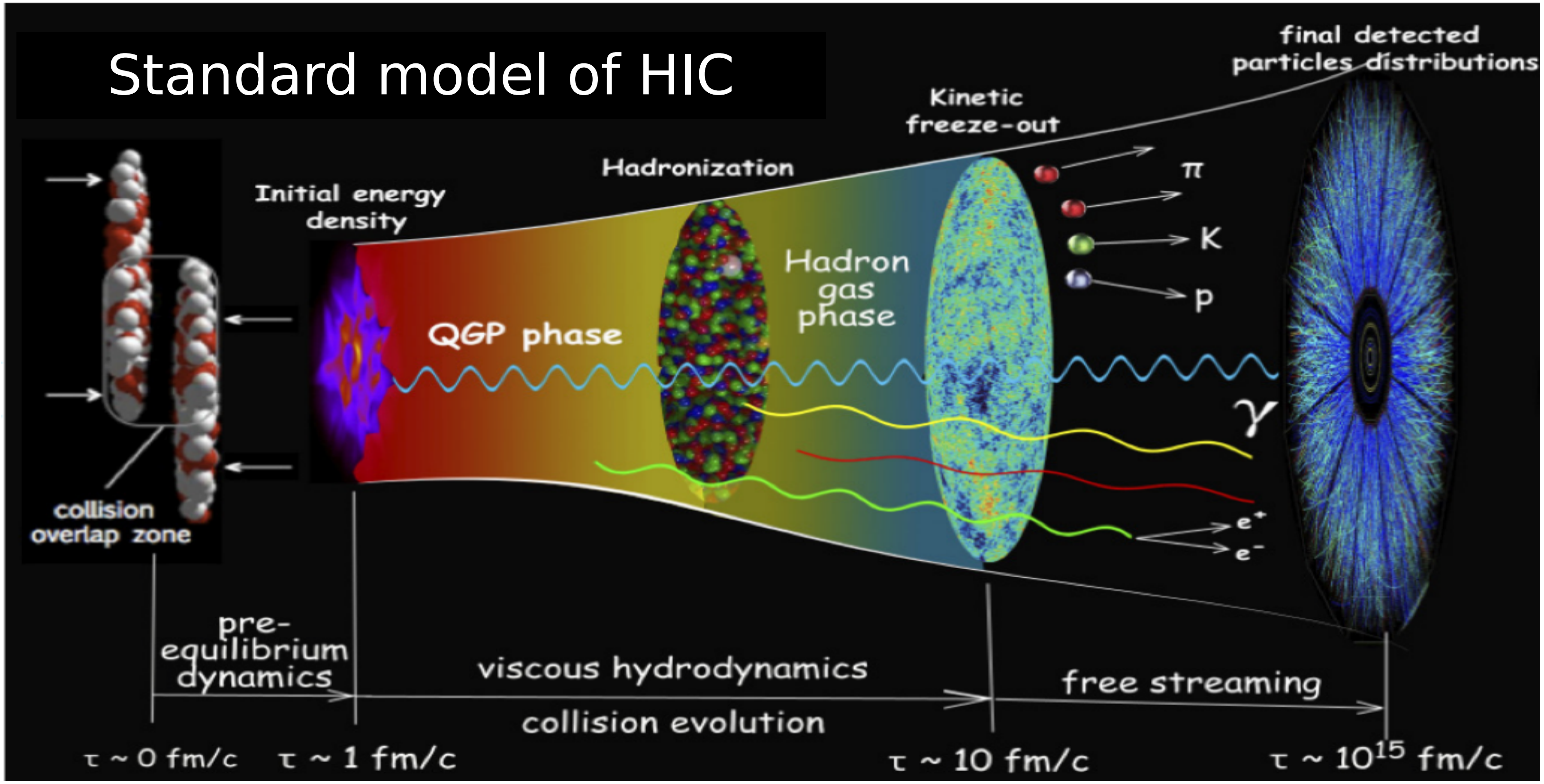




How can one understand the complex dynamics of a heavy-ion collisions?

Heavy-ion collisions

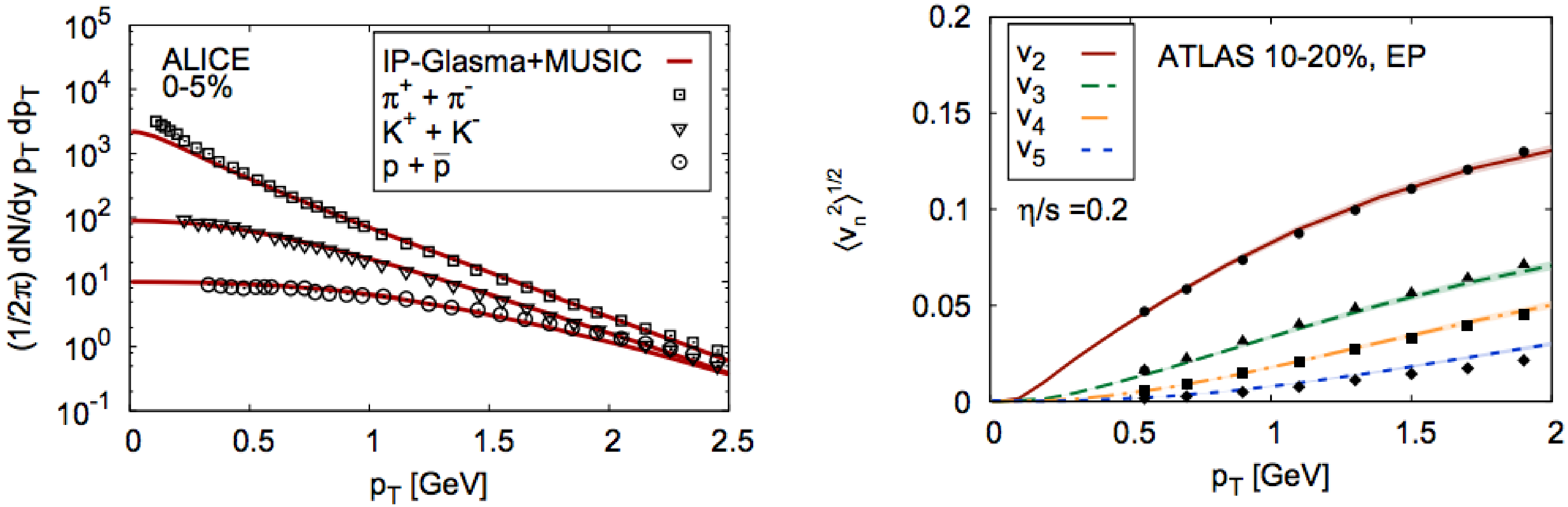
Conjectured space-time evolution of a heavy-collision based on phenomenological models and experimental information



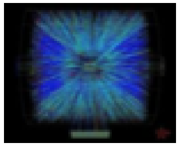
(c. f. U.Heinz, J.Phys.Conf.Ser. 455 (2013) 012044)

Heavy-ion collisions

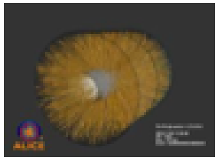
Hydrodynamic simulations versus experiment



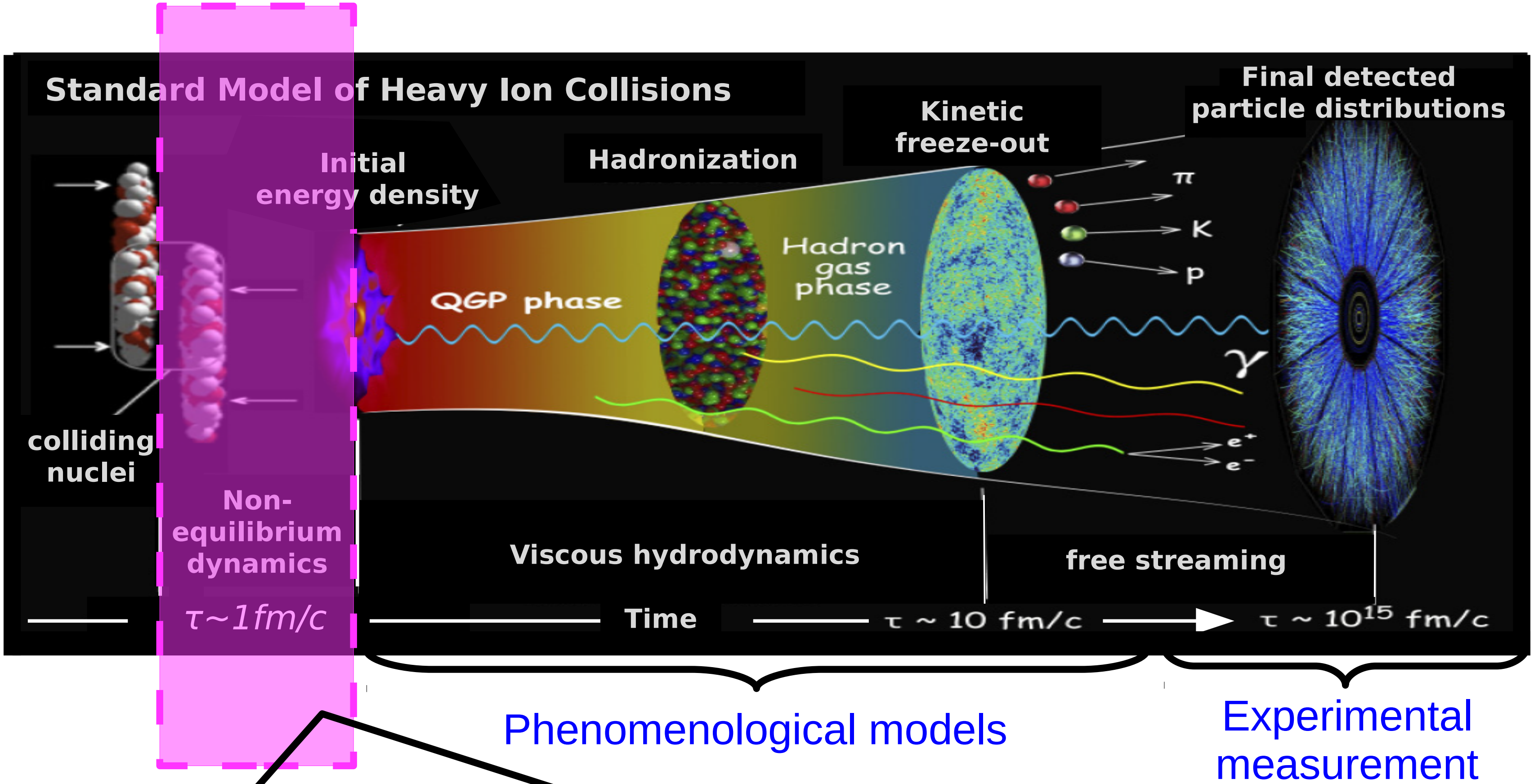
Schenke et al. PRL 110 (2013) 012302



A large variety of data at RHIC and LHC can be explained based on this standard model



The thermalization problem



When and to what extent is a thermalized QGP achieved?
How does this happen?

Thermalization process

Progress in a first-principle understanding from two limiting cases

Holographic thermalization:

a) strong coupling? *Heller, Janik, Witaszczyk; Chesler, Yaffe ...*

Sizeable anisotropy at transition to hydrodynamic regime

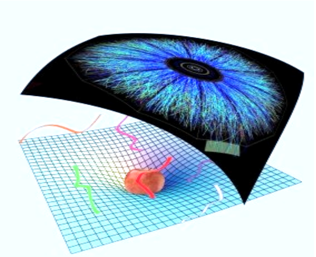


Fig. from strings.net.technion.ac.il

Turbulent thermalization:

b) weak coupling but highly occupied? *CGC: McLerran, Venugopalan ...*

Energy density of gluons with typical momentum Q_s (at time ~$1/Q_s$)

$$\epsilon \sim \frac{Q_s^4}{\alpha_s}$$ i.e. 'occupation numbers' $$n(p \lesssim Q_s) \sim \frac{1}{\alpha_s}$$

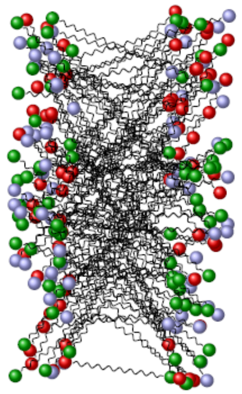


Fig. by T. Epelbaum

Weak coupling out-of-equilibrium methods

Classical-statistical field theory

Whenever the occupancy/field amplitudes are large (f >> 1) a description in terms of classical field equations of motion is applicable

$$D_\mu F^{\mu\nu} = J^\nu$$

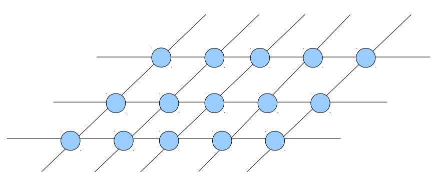
→ Can be solved numerically for a discretized space-time using standard lattice techniques

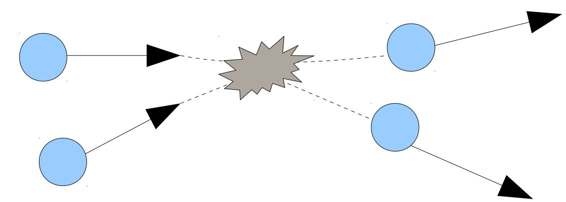
Kinetic theory

Whenever the occupancy becomes less than (f <1/λ) a description in terms of quasi particle excitations should also be applicable

$$\partial_t f(t,p) = C[f](t,p)$$

→ Can study the effect of individual processes (e.g. 2 ↔2 or 2↔3 scattering)





$f(t,p) \sim 1/\lambda$

('strongly interacting system')

$1/\lambda > f(t,p) > 1$

('classical particles')

$f(t,p) \sim 1$

('quantum')

→ Overlap in the range of applicability

Thermalization process

Initial state:
Far from equilibrium

Non-equilibrium dynamics

Final state:
Thermal equilibrium

How is thermal equilibrium achieved?

Thermalization process

Non-equilibrium phenomena may be shared by a large class of strongly correlated many-body systems

I) Thermalization in scalar field theory (c.f. Cosmology)

(Micha, Tkachev PRD 70 (2004) 043538)
(Berges, Boguslavski,SS, Venugopalan arXiv:1312.5216)

II) Thermalization in Yang-Mills theory in Minkowski space

(Berges,SS,Sexty PRD 86 (2012) 074006; SS PRD 86 (2012) 065008)

III) Thermalization in heavy-ion collisions at ultra-relativistic energies

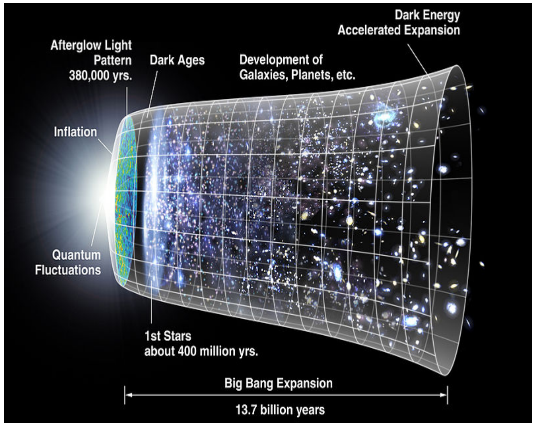
(Berges,Boguslavski,SS,Venugopalan arXiv:1303.5650, arXiv:1311.3005)

Scalar theory – Reheating model

Scalar field theory (λΦ4); Small coupling $\lambda = 10^{-8}$

$$S[\varphi] = \int d^4x \left(\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{\lambda}{24}\varphi^4 \right)$$

Simplest model for thermalization of the early universe *(Micha, Tkachev PRD 70 (2004) 043538)*



Initial conditions (e.g. at the end of inflation):

Homogenous background field (condensate) $\Phi_0 \sim 1/\sqrt{\lambda}$ + vacuum fluctuations

Coupling constant is typically very small

$$\lambda \sim 10^{-8}$$

The initial field amplitude of the inflation field is large

$$\phi \sim 1/\sqrt{\lambda}$$

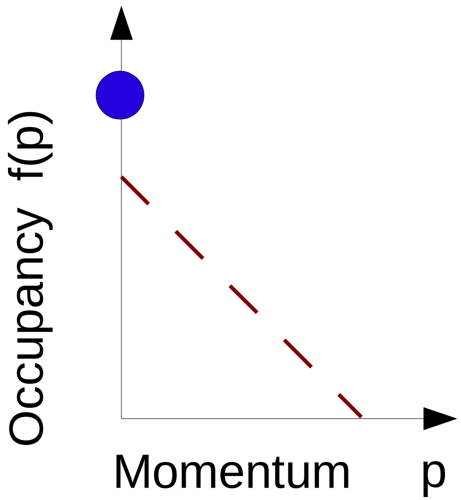
→ ***Weakly coupled but strongly interacting***

What happens during thermalization?

Initially most energy is stored in the background field

In thermal equilibrium energy is mostly carried by momentum modes ~T

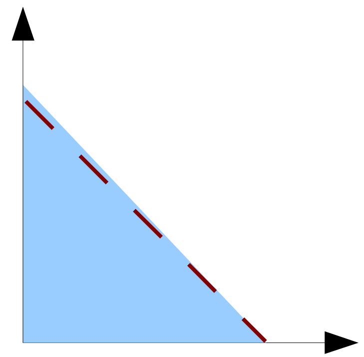
Initial state –
Far from equilibrium



???

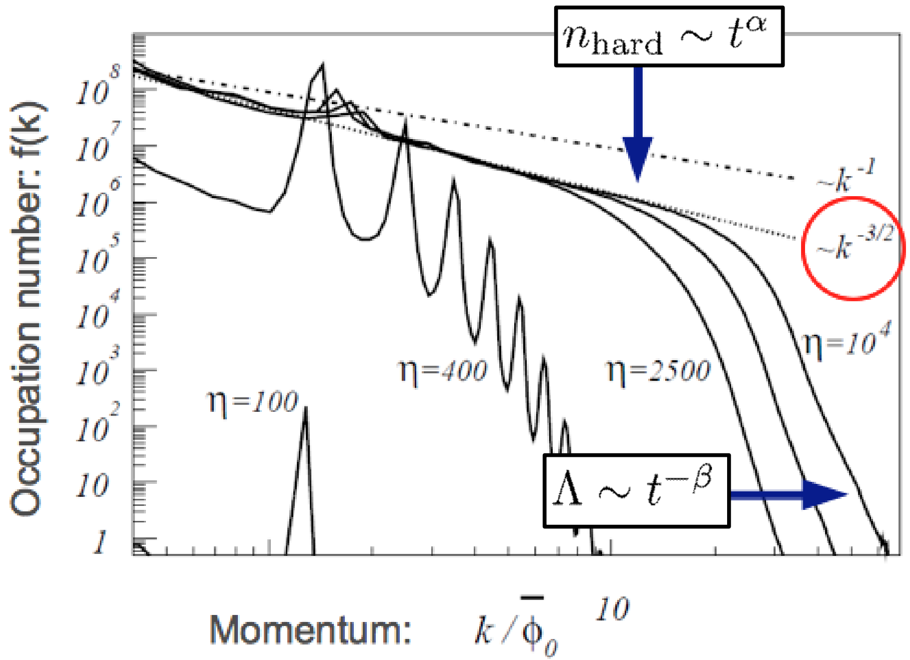
Energy transport towards Ultra-Violet

Thermal
equilibrium

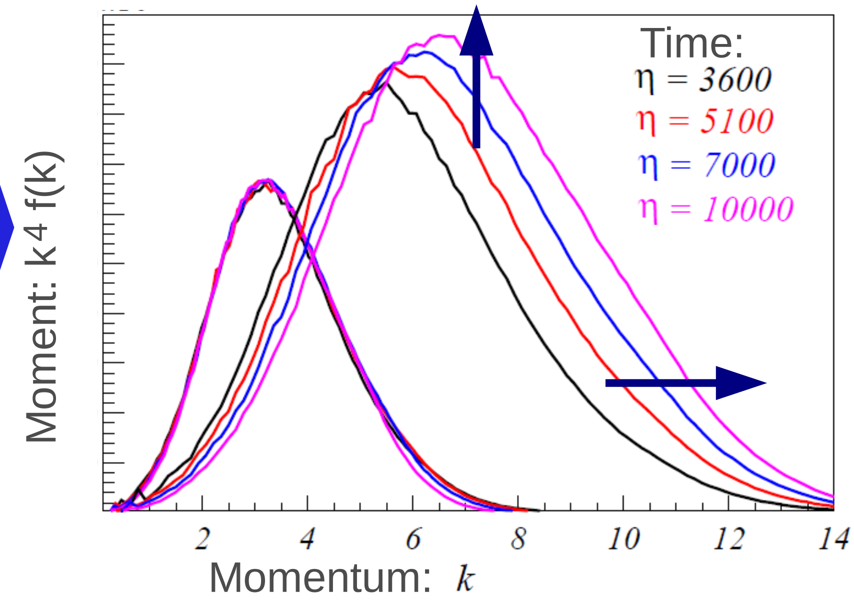


Thermalization process

Non-thermal fixed point



Self-similar evolution



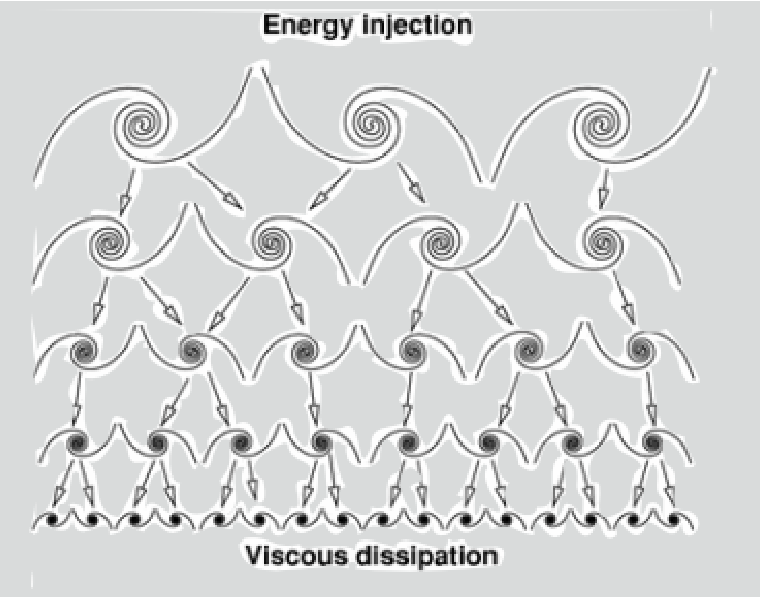
- The evolution becomes ***self-similar*** $f(p,t) = t^{\alpha}\ f_S(t^{\beta}p)$

- The thermalization process is described by a ***quasi-stationary evolution*** with ***scaling exponents*** Dynamic: α=-4/5 β=-1/5 Spectral: κ=-3/2

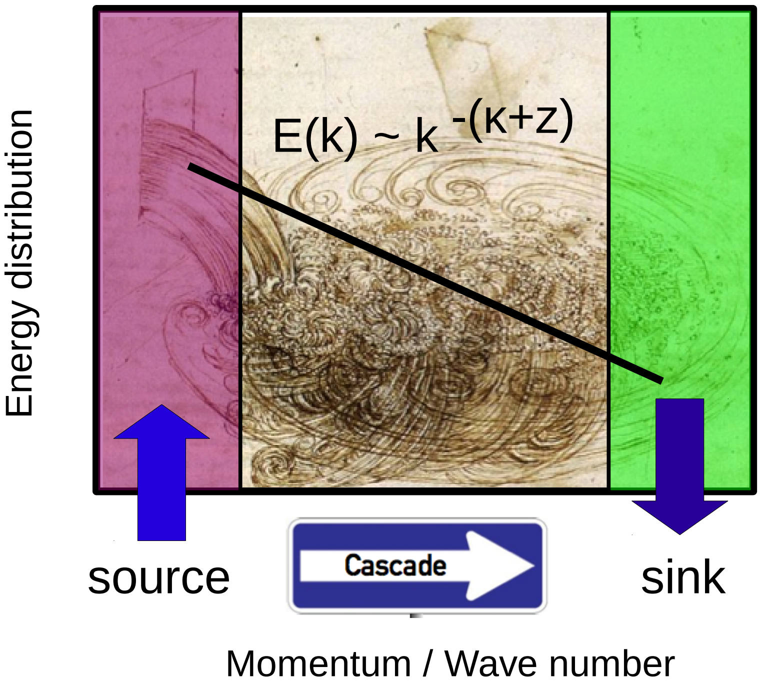
(Micha, Tkachev PRD 70 (2004) 043538)

Turbulent thermalization – Classical picture of wave turbulence

Richardson cascade



Kolmogorov spectra



Uriel Frisch, *“Turbulence. The Legacy of A. N. Kolmogorov.”*

Zakharov, V. E.; L'vov, V. S.; Falkovich, G, *“Kolmogorov spectra of turbulence 1. Wave turbulence.”*

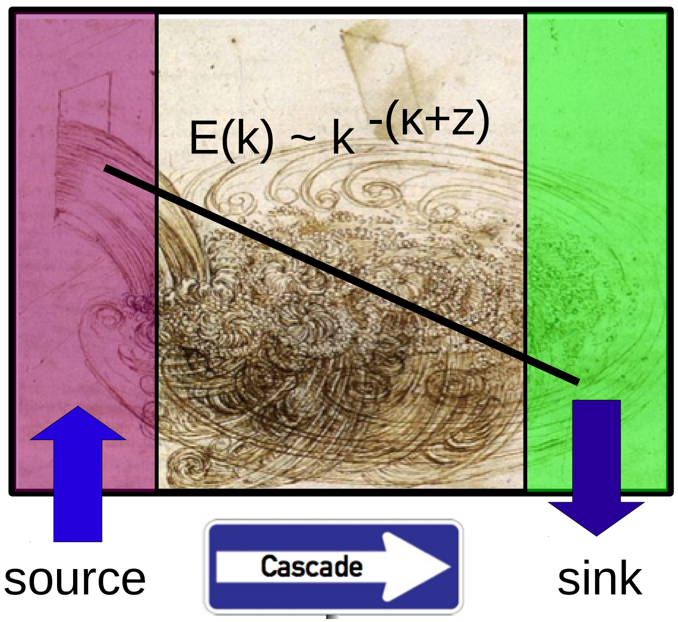
- ***Stationary scaling solution*** associated with scale invariant energy flux

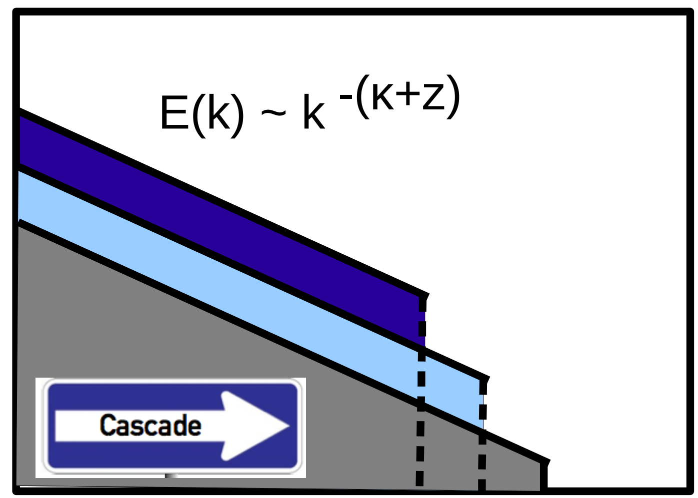
Turbulent thermalization – Wave turbulence in closed systems

"Driven" Turbulence – Kolmogorov wave turbulence

vs.

"Free" Turbulence – Turbulent Thermalization





closed system

- ***Stationary scaling solution*** associated with scale invariant energy flux

- ***Quasi-stationary scaling solution***
- ***Self-similar time evolution*** associated with energy transport towards the ultra-violet

Kinetic interpretation

- Search for ***self-similar scaling solutions***

$$f(p,t) = t^{\alpha}\ f_S(t^{\beta} p)$$

of the Boltzmann equation

$$\partial_t f(p,t) = C[f](p,t) \xrightarrow[(f \gg 1)]{scale\ \ invariance} C[f](p,t) = t^{\mu} C[f_S](t^{\beta} p)$$

→ Boltzmann equation reduces to a fixed point equation and a scaling relation

$$\alpha f_S(p) + \beta \partial_p f_S(p) = C[f_S](p) \qquad \alpha - 1 = \mu(\alpha, \beta)$$

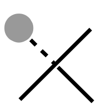
(Cosmology: Micha, Tkachev PRD 70 (2004) 043538)

Turbulent thermalization

- The dynamic ***scaling exponents*** are uniquely determined by

Canonical scaling of the collision integral **+** *Conservation laws* → ***Universality far from equilibrium***

- Classification scheme for relativistic field theories *(Micha, Tkachev)*

Interaction		***Λ evolution (Exponent α)***	***Occupancy evolution (Exponent β)***	***Spectral Shape (Exponent κ)***
	***2<->1+soft***	***-1/5***	***-4/5***	***3/2***
	***2<->2***	***-1/7****	***-4/7****	*4/3*, ***5/3***

“free” cascade (Λ and occupancy evolution, 2<->2) — ***stationary cascade*** (spectral shape, 2<->2)

→ ***Scalar theory:*** Turbulent cascade is driven by ***2<->(1+soft)*** interaction

(c.f. Kurkela, Moore for SU(Nc) gauge theory)*

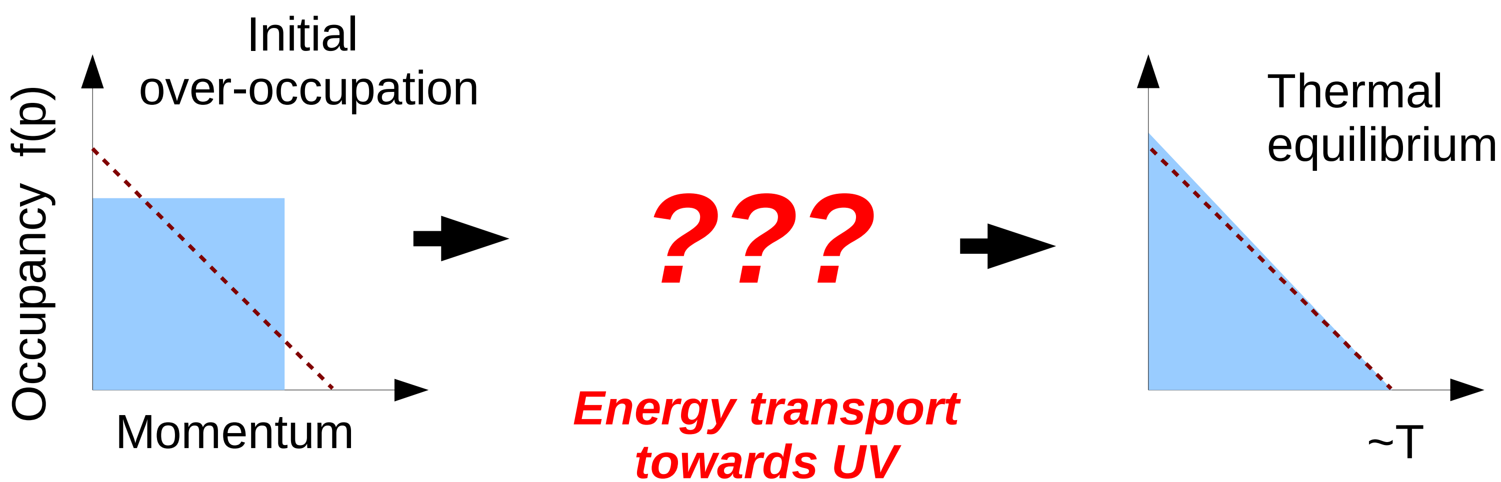
Generic phenomenon?

- Consider e.g. initial conditions without a condensate

$$f(t_0,p)=\frac{n_0}{\lambda}\theta(Q-p)$$

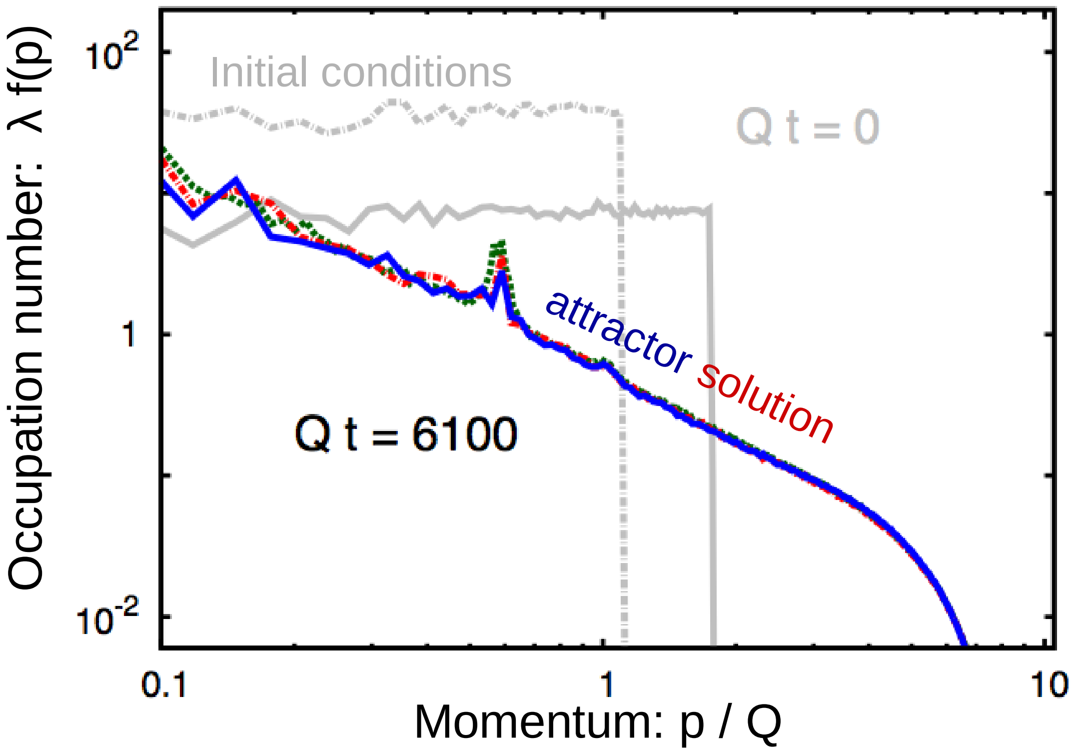
n_0 controls initial over-occupancy

→ *Thermalization process remains essentially the same*



(Berges,Boguslavski,SS,Venugopalan arXiv:1312.5216)

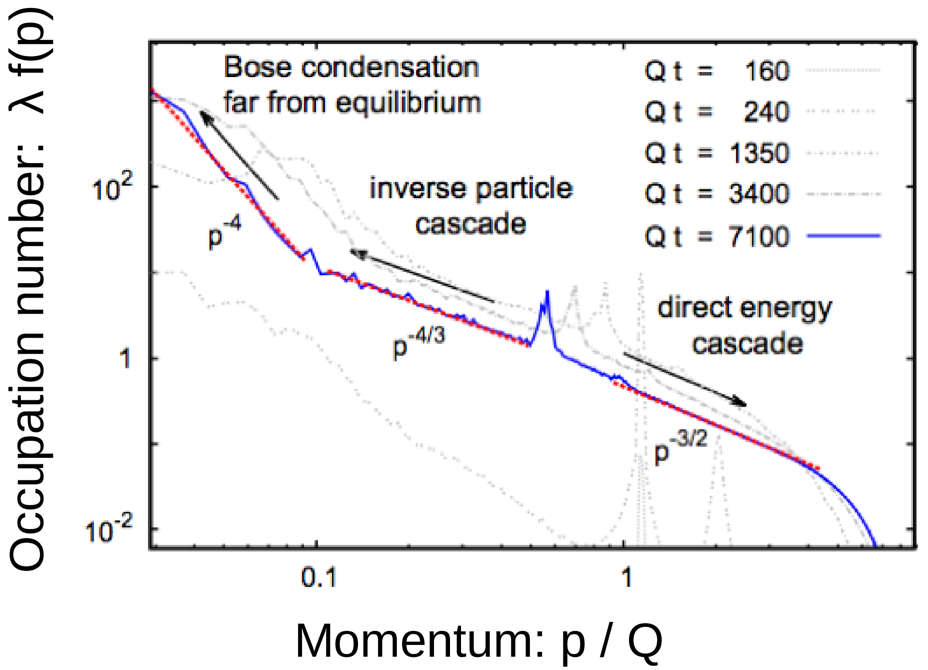
Independence of Initial conditions

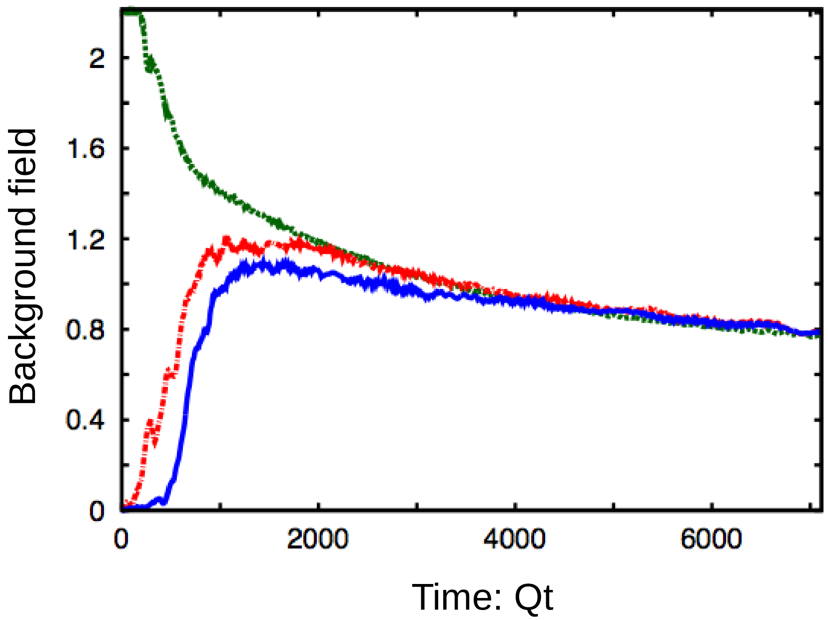


- The turbulent scaling behavior is a property of the thermalization process – ***independent of the underlying initial conditions***
- An ***effective memory loss*** occurs ***already at the early stages*** of the thermalization process

(Berges,Boguslavski,SS,Venugopalan arXiv:1312.5216)

Bose-Condensation far from equilibrium



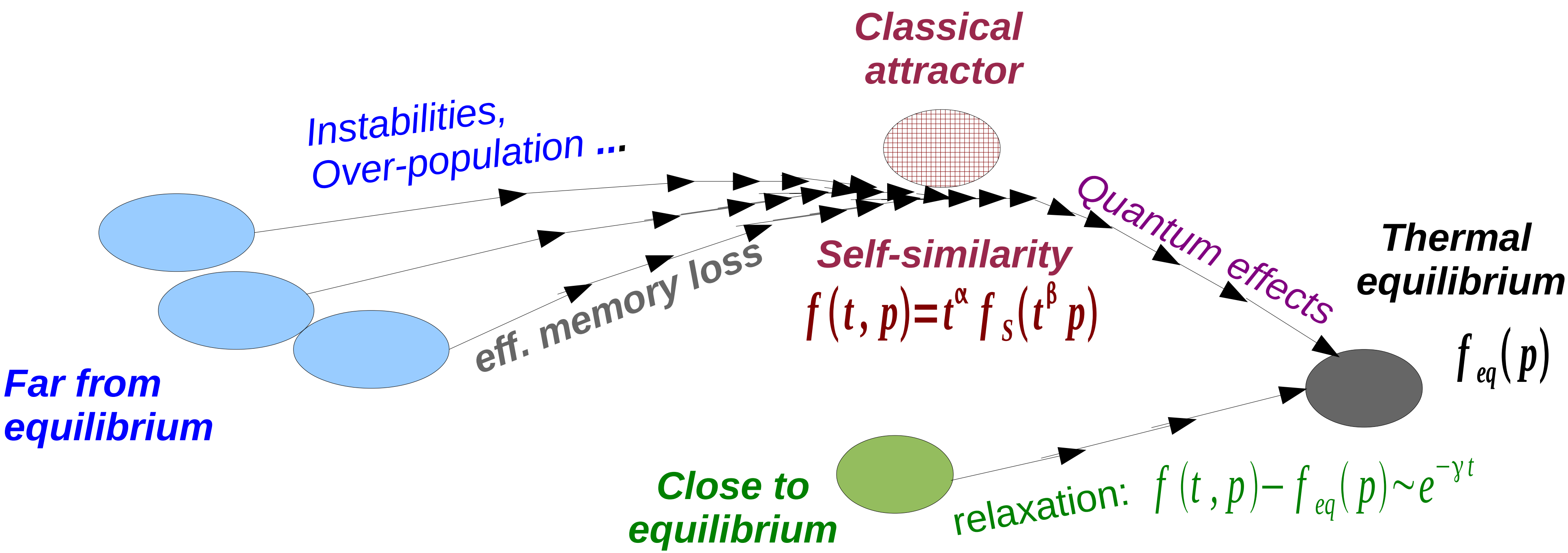


→ Dynamical formation of macroscopic zero mode (Bose condensation) even the though the system is in the symmetric phase

(Berges, Sexty PRL 108 (2012) 161601 Berges,Boguslavski,SS,Venugopalan arXiv:1312.5216)

Turbulent thermalization

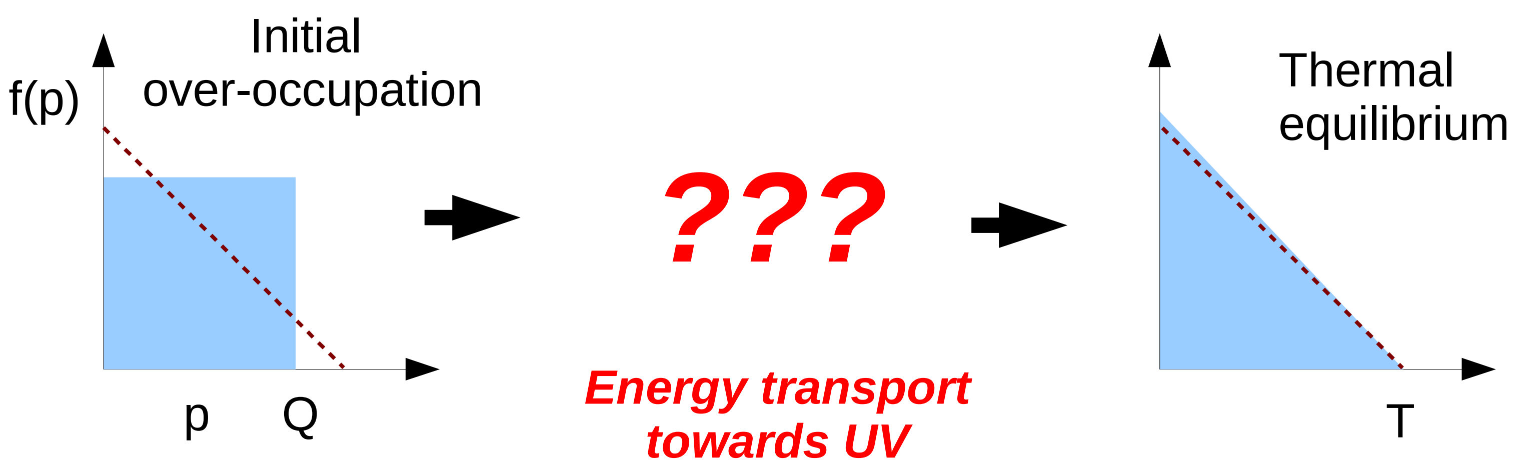
Thermalization for a system far from equilibrium proceeds as a ***self-similar evolution*** associated to the presence of a ***non-thermal fixed point***



How does this picture apply to non-Abelian gauge theories?
Does it hold for relativistic heavy-ion collisions?

Non-abelian plasma in Minkowski space

- Consider homegenous and ***isotropic*** systems which are initially ***highly occupied*** and initially characterized by a single momentum scale Q



How does thermalization proceed? Turbulent attractor?
What are the relevant kinetic processes?

(c.f. Kurkela, Moore JHEP 1112 (2011) 044; Blaizot et al. Nucl.Phys. A873 (2012) 68-80)

Definition of occupation number

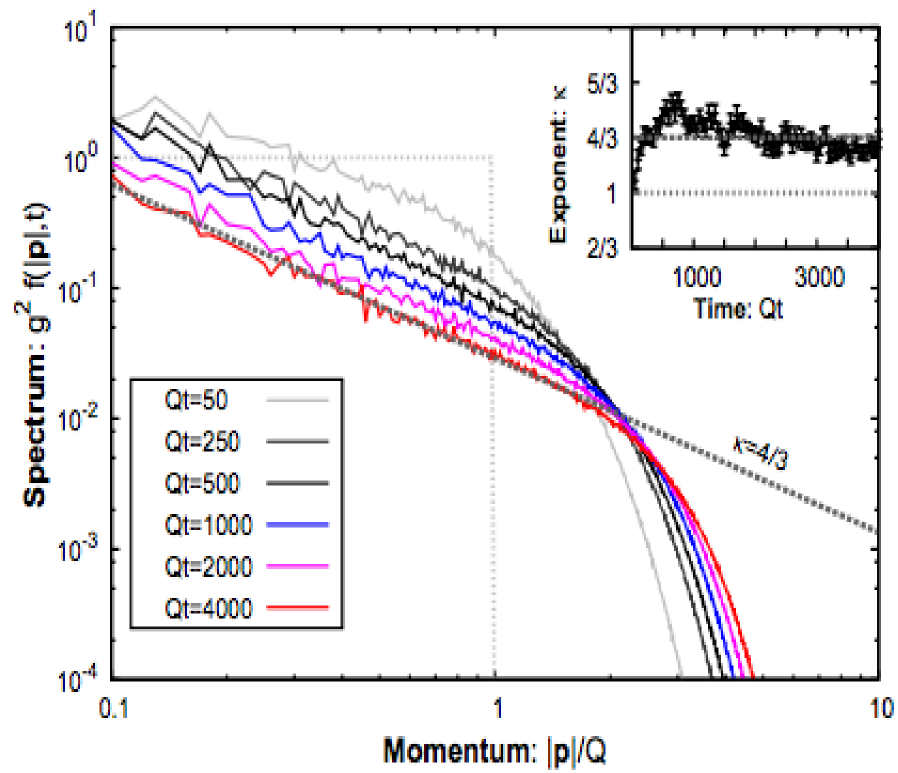
- Single particle distribution ***is a gauge dependent quantity*** but facilitates comparison with kinetic theory.

- Chose ***temporal axial + Coulomb type gauge*** to fix the gauge freedom

$$A_t = 0 \qquad \nabla A\Big|_t = 0$$

- Define occupation number from ***equal time correlation functions***

$$f(p,t) = \left\langle \left| \xi_\mu^{(\lambda)k}(t)\, \overleftrightarrow{\partial_t}\, A_a^\mu(t,p) \right|^2 \right\rangle_{\text{(Coul. gauge)}}$$



(see e.g. Kurkela, Moore PRD 86, (2012) 056008; SS PRD86 (2012) 065008; Berges,Boguslavski,SS, Venugopalan PRD89 (2014) 114007)

Self-similarity

- Evolution at late times shows a ***self-similar*** behavior

$$f(p,t)=(Qt)^{\alpha} f_S((Qt)^{\beta} p)$$

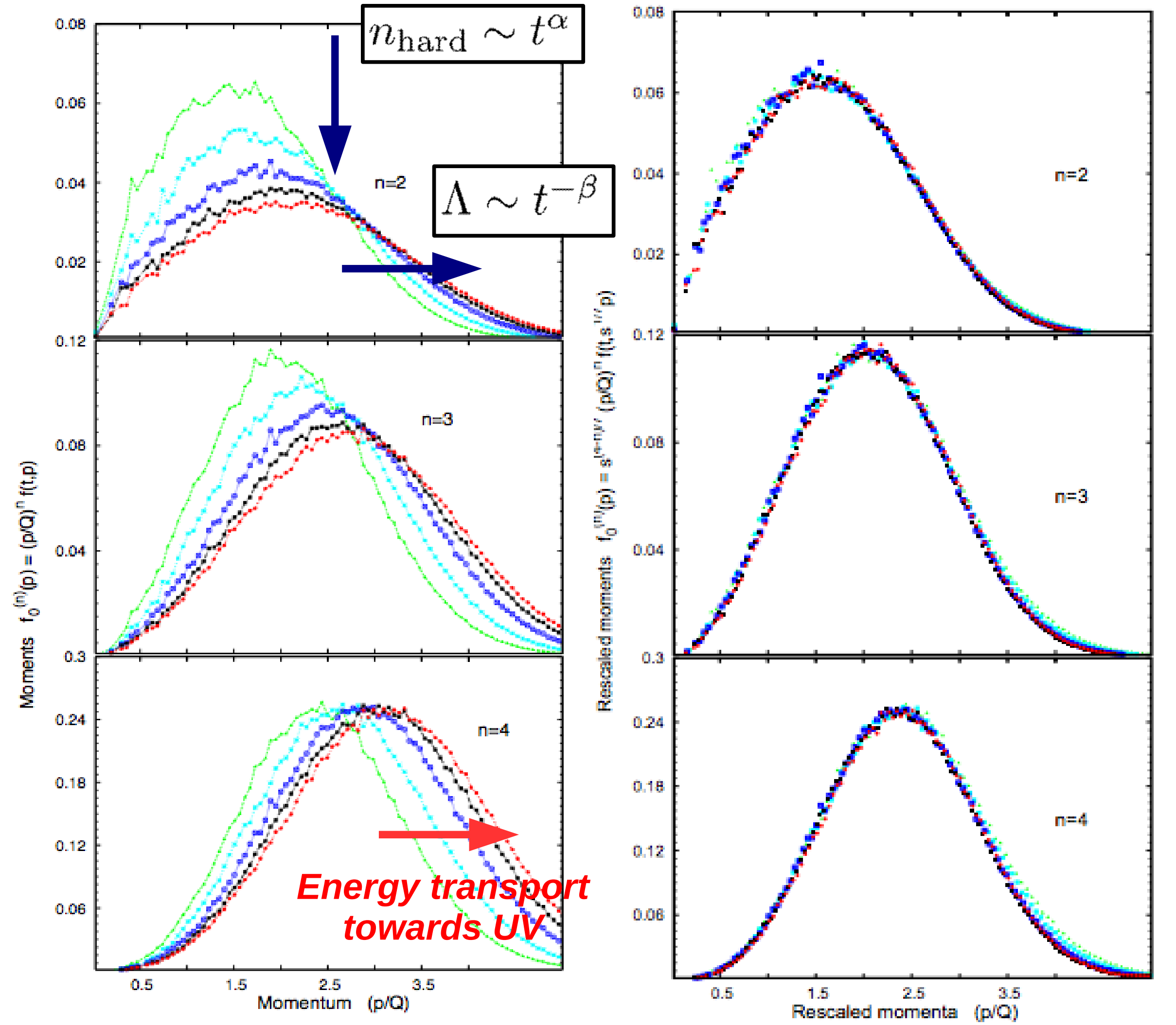
with dynamical scaling exponents

$\alpha = -4/7$

$\beta = -1/7$

consistent with ***elastic & inelastic scattering*** processes

(c.f. Kurkela, Moore JHEP 1112 (2011) 044; Blaizot et al. Nucl.Phys. A873 (2012) 68-80)



(SS PRD 86 (2012) 065008; Kurkela, Moore PRD 86, (2012) 056008)

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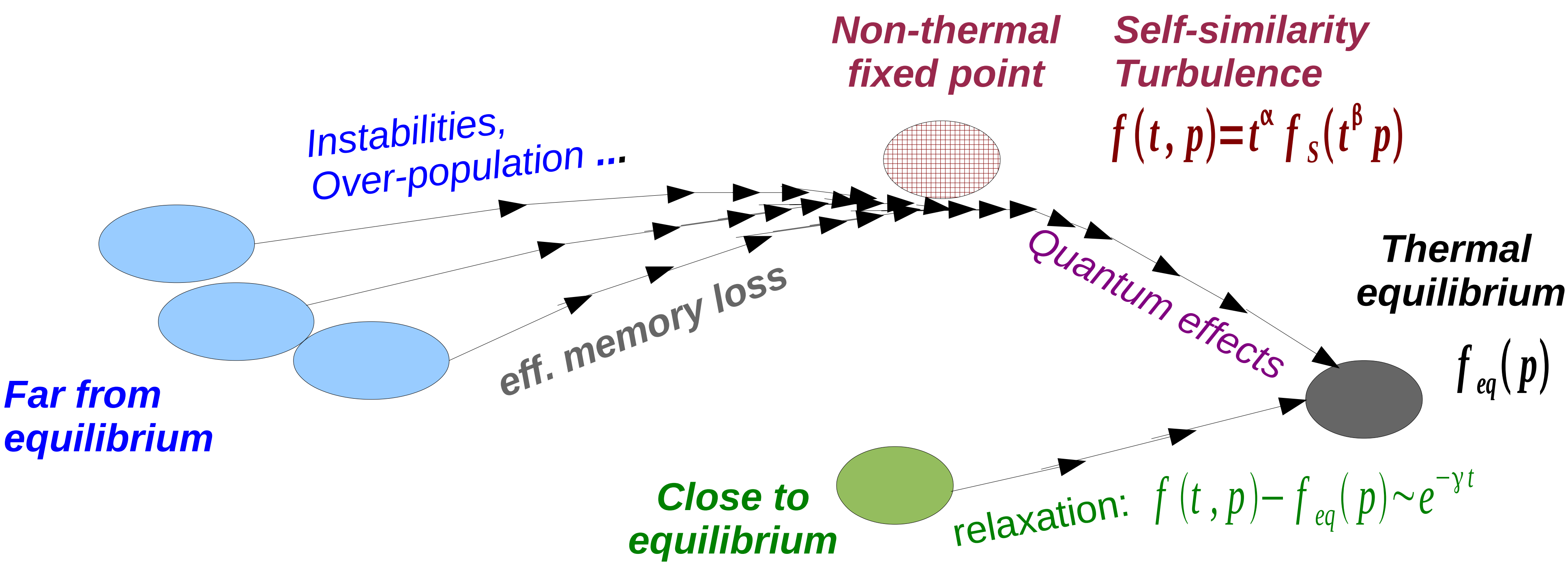

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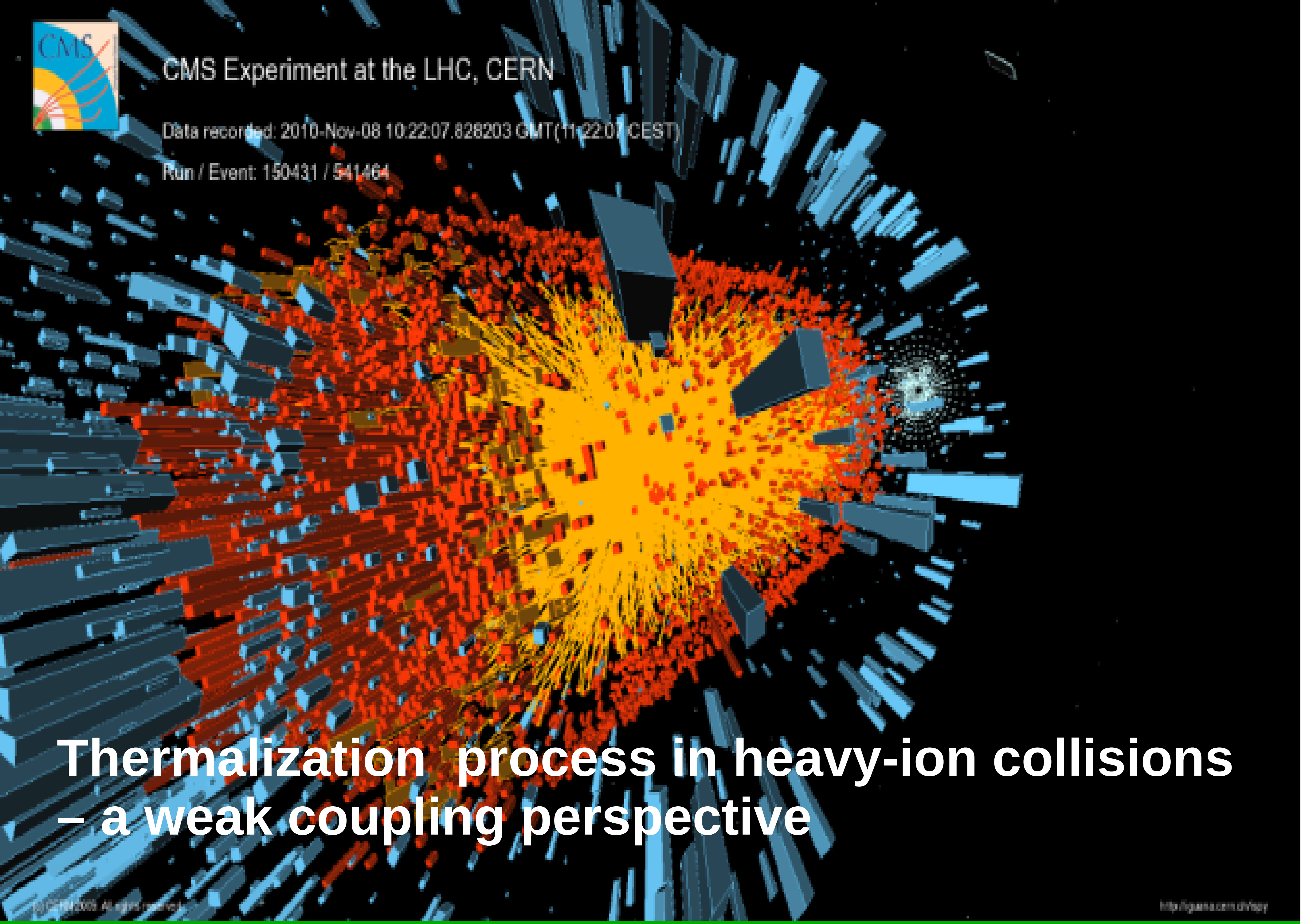
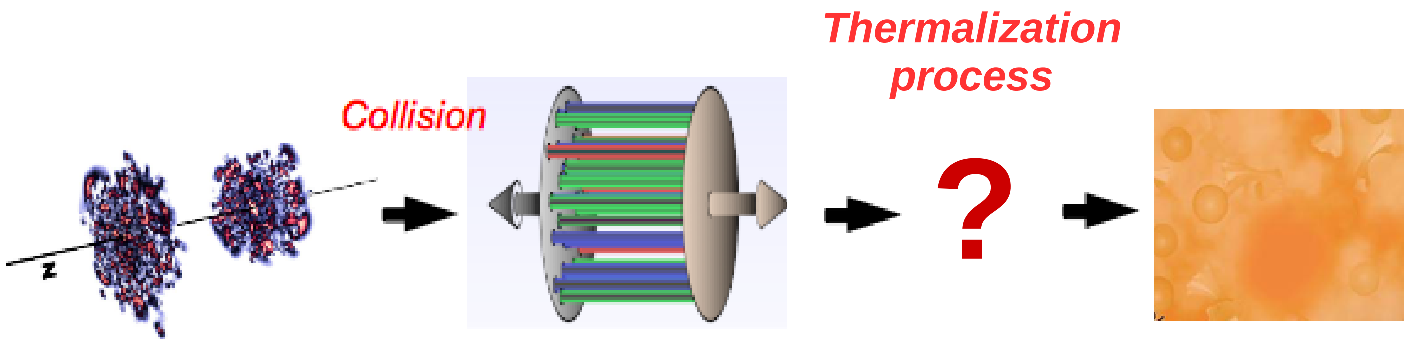


Thermalization process in heavy-ion collisions – a weak coupling perspective

General picture at weak coupling



— Time ————→

Colliding nuclei

Gluons produced in highly anisotropic state *'Glasma'*

Thermalized QGP

Coupling constant assumed to be small

$$\alpha_s(Q) \ll 1$$

Many gluons prodcued in the collision

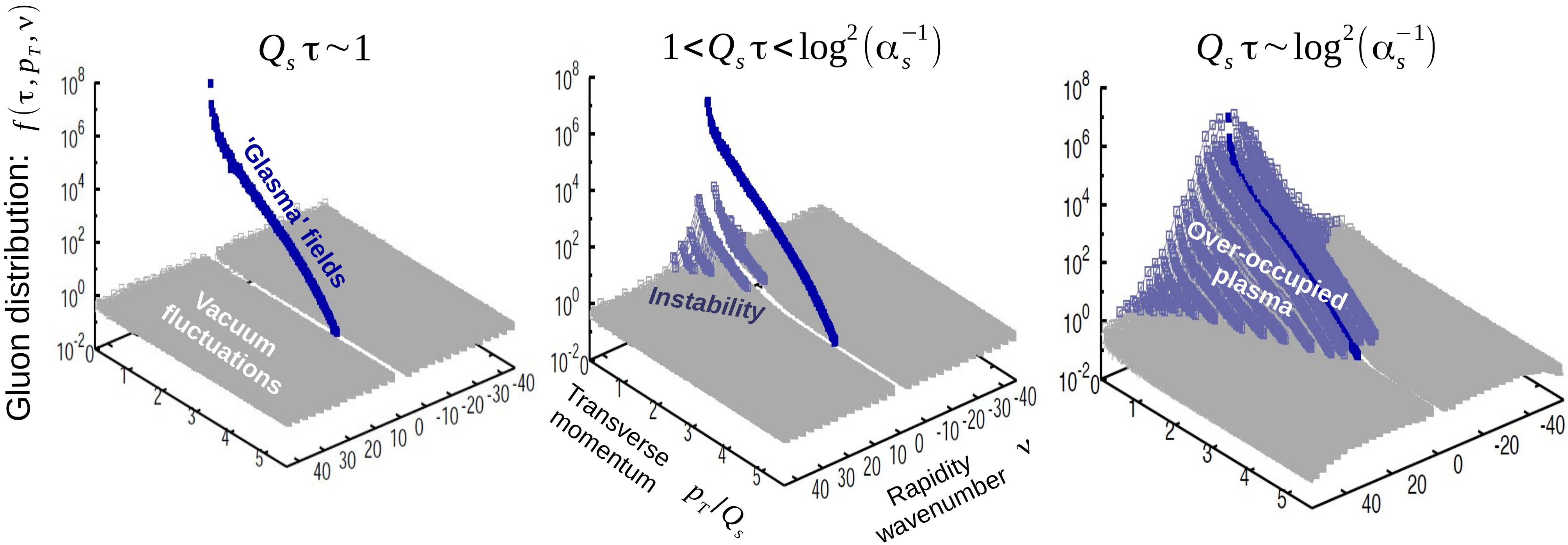
$$f(p) \sim 1/\alpha_s$$

→ ***Weakly coupled but strongly interacting***

Early time dynamics

Initial state is highly anisotropic → Plasma instabilities lead to exponential growth of low momentum modes *(c.f. Mrowczynski, Romatschke, Strickland, Rebhan, Attems , Venugopalan, Epelbaum, Gelis, Fukushima, Berges, Sexty ...)*

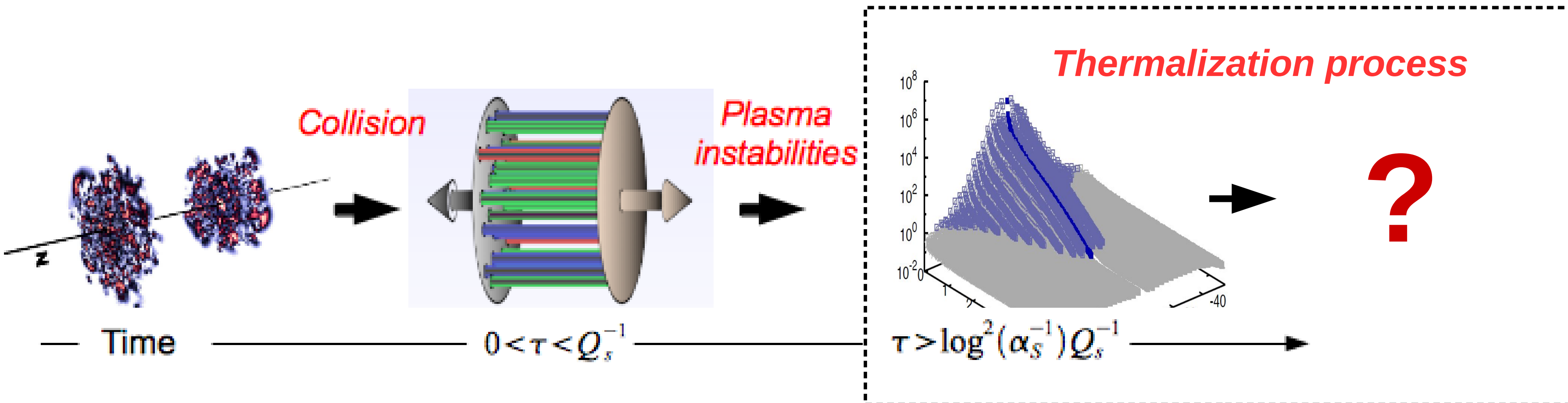
→ ***Over-occupied plasma*** $f(p \lesssim Q_s) \sim 1/\alpha_s$ ***formed on a time scale*** $\tau \sim Q_s^{-1} \log^2(\alpha_s^{-1})$



(Berges,Schenke,SS, Venugopalan work in progress)

Thermalization process at weak coupling

System is still far from equilibrium at times $\tau_0 = 1/Q_s \ln^2(1/\alpha_s)$



Competition between interactions and the longitudinal expansion, may render the system ***anisotropic on large time scales***

Longitudinal Expansion:
Red-shift of longitudinal momenta
→ Increase of anisotropy
→ Dilution of the plasma

Interactions:
Momentum broadening
→ Decrease of anisotropy

Thermalization scenarios

- Different scenarios of how thermalization proceeds have been proposed in the literature

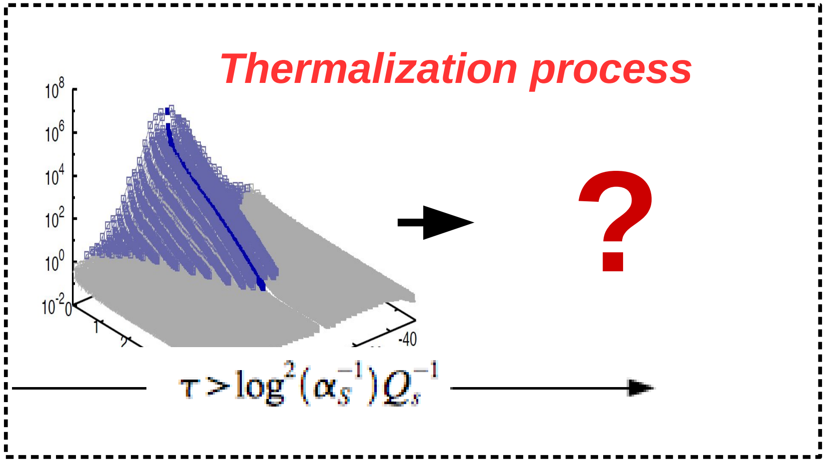
Baier et al. (BMSS),
PLB 502 (2001) 51-58 → ***Elastic + inelastic scattering***

Boedeker (BD),
JHEP 0510 (2005) 092 → ***Plasma instabilities***

Kurkela, Moore (KM),
JHEP 1111 (2011) 120 → ***Plasma instabilities***

→ Difference arises from the treatment of soft (non-perturbative) physics of modes below the Debye scale.

Thermalization of the over-occupied QGP



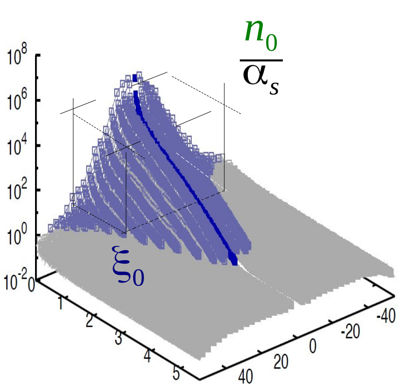
Classical regime can be studied non-perturbatively within classical-statistical lattice simulations

Study thermalization process for a variety of different initial conditions which describe the the over-occupied plasma at initial time $\tau_0 = 1/Q_s \ln^2(1/\alpha_s)$

Over-occupation

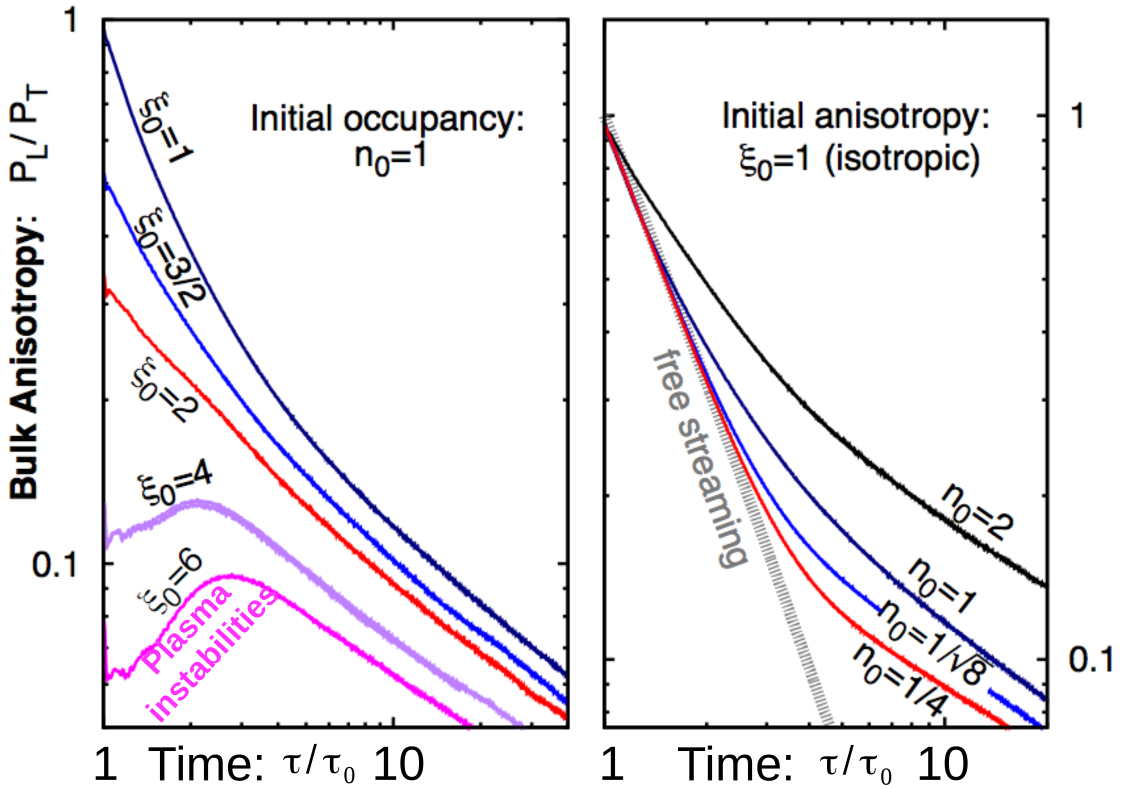
Momentum space anisotropy

$$f(p_T, p_Z, \tau_0) = \frac{n_0}{\alpha_s} \theta\left(Q_s - \sqrt{p_T^2 + \xi_0^2 p_Z^2}\right)$$



(Berges, Boguslavski, SS, Venugopalan PRD 89 074011 & arXiv:1311.3005)

Lattice results – Bulk anisotropy



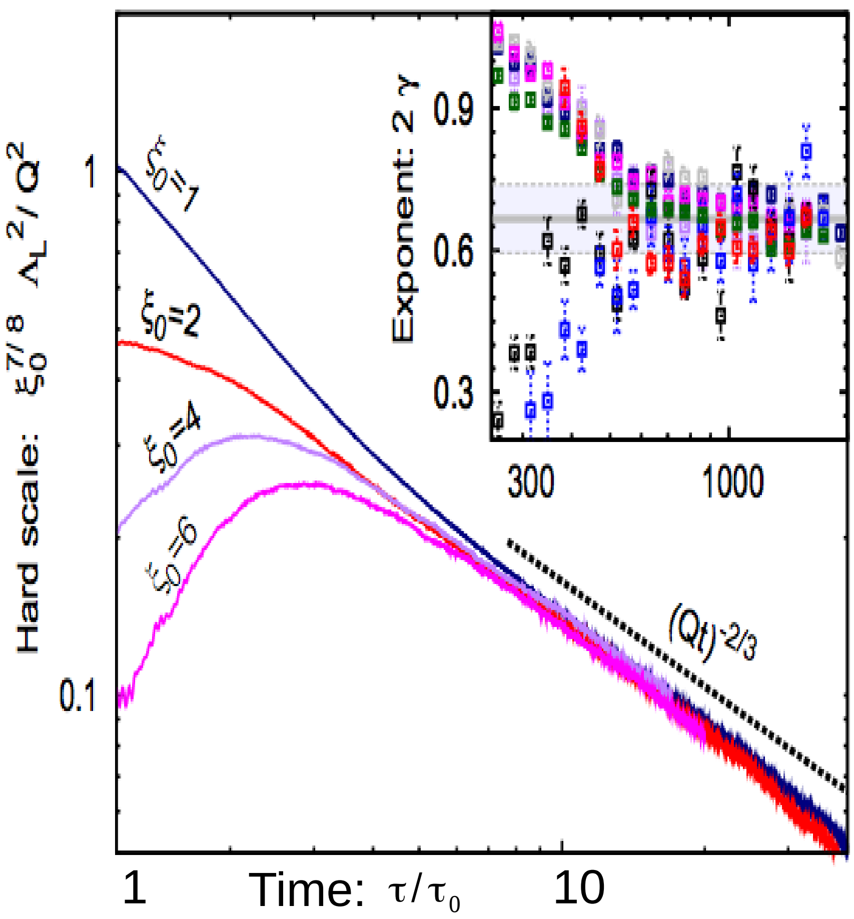
- Competition between interactions and longitudinal expansion leads to an ***increase of the anisotropy.***
- Nevertheless the system remains ***significantly interacting*** throughout the entire evolution.
- The evolution becomes ***insensitive to the initial conditions*** and exhibits a ***universal scaling behavio***r at late times.

n_0 controls initial over-occupancy

ξ_0 controls initial anisotropy

(Berges, Boguslavski, SS, Venugopalan PRD 89 074011 & arXiv:1311.3005)

Universal Scaling



- The typical ***longitudinal momentum*** of hard excitations exhibits a ***universal scaling*** behavior

$$\Lambda_L^2/Q^2 \sim (Qt)^{-2\gamma}$$

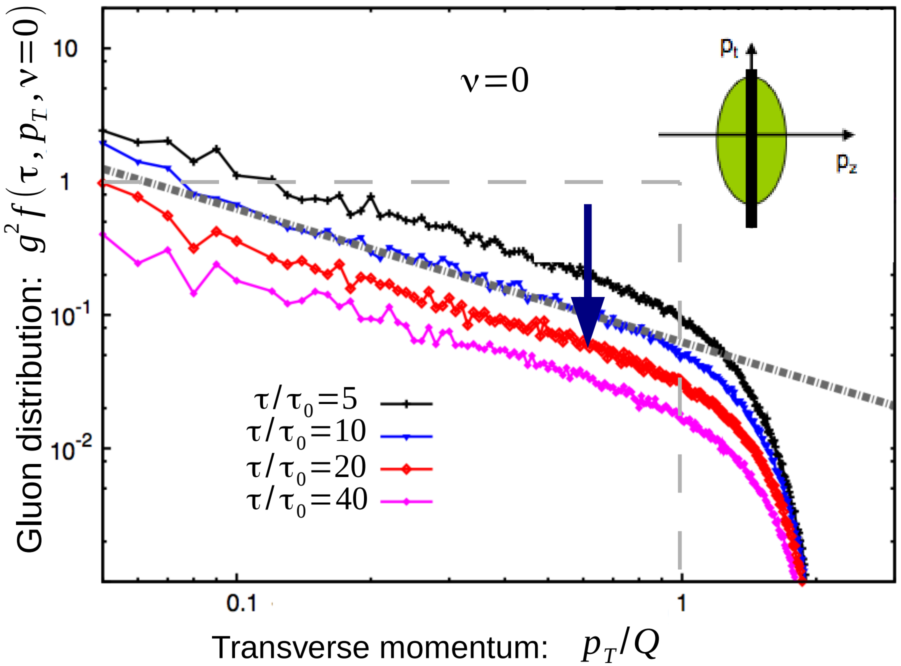
$$\boxed{2\gamma \quad = \quad 0.67 \pm 0.07}$$

- The typical ***transverse momentum*** of hard excitations remains approximately ***constant***

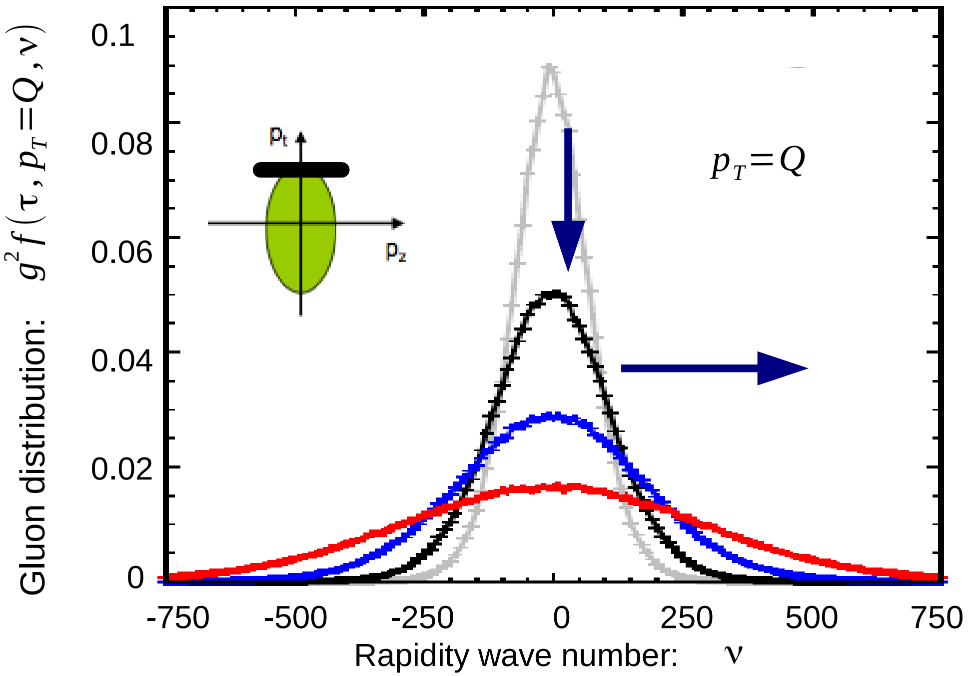
$$\Lambda_T^2/Q^2 \sim (Qt)^{-2\beta} \qquad \boxed{2\beta \simeq 0}$$

Single particle spectra

Transverse spectrum



Longitudinal spectrum



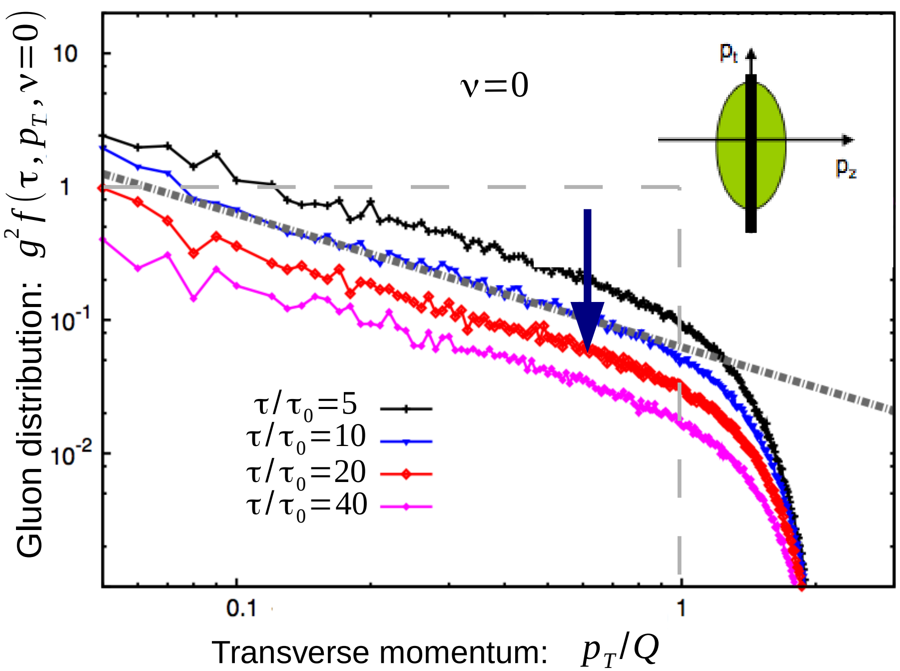
Transverse spectrum quickly approaches 'thermal' like T/p_T shape, with decreasing amplitude

Significant momentum broadening in the longitudinal direction observed.

(Berges, Boguslavski, SS, Venugopalan PRD 89 074011 & arXiv:1311.3005)

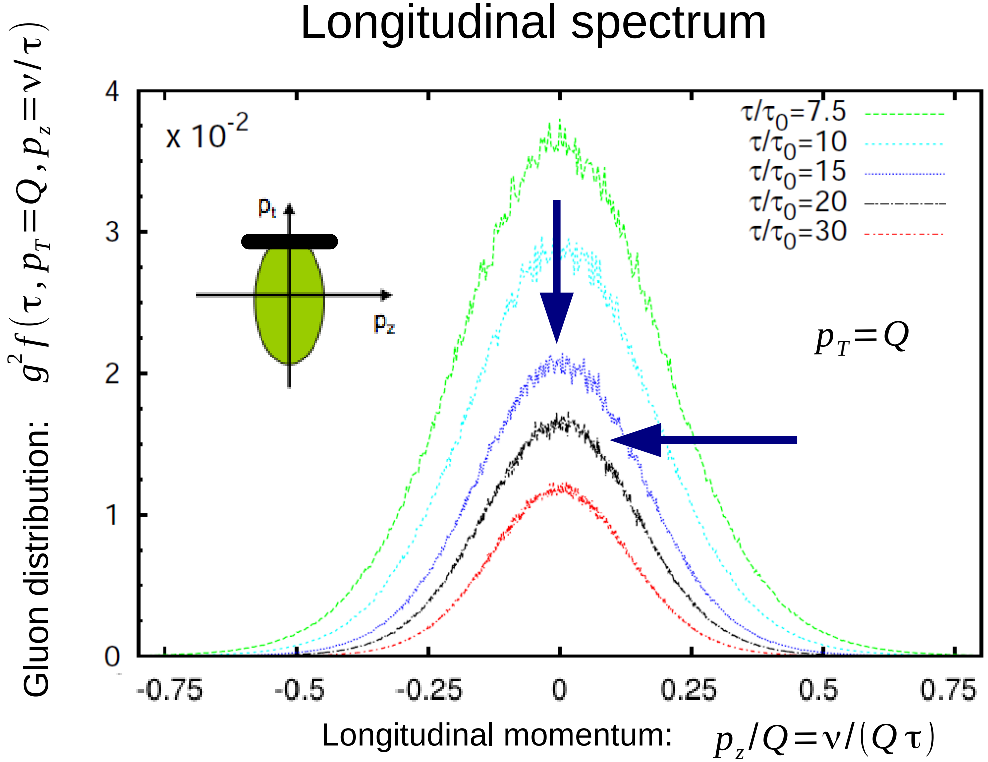
Single particle spectra

Transverse spectrum



Transverse spectrum quickly approaches 'thermal' like T/p_T shape, with decreasing amplitude

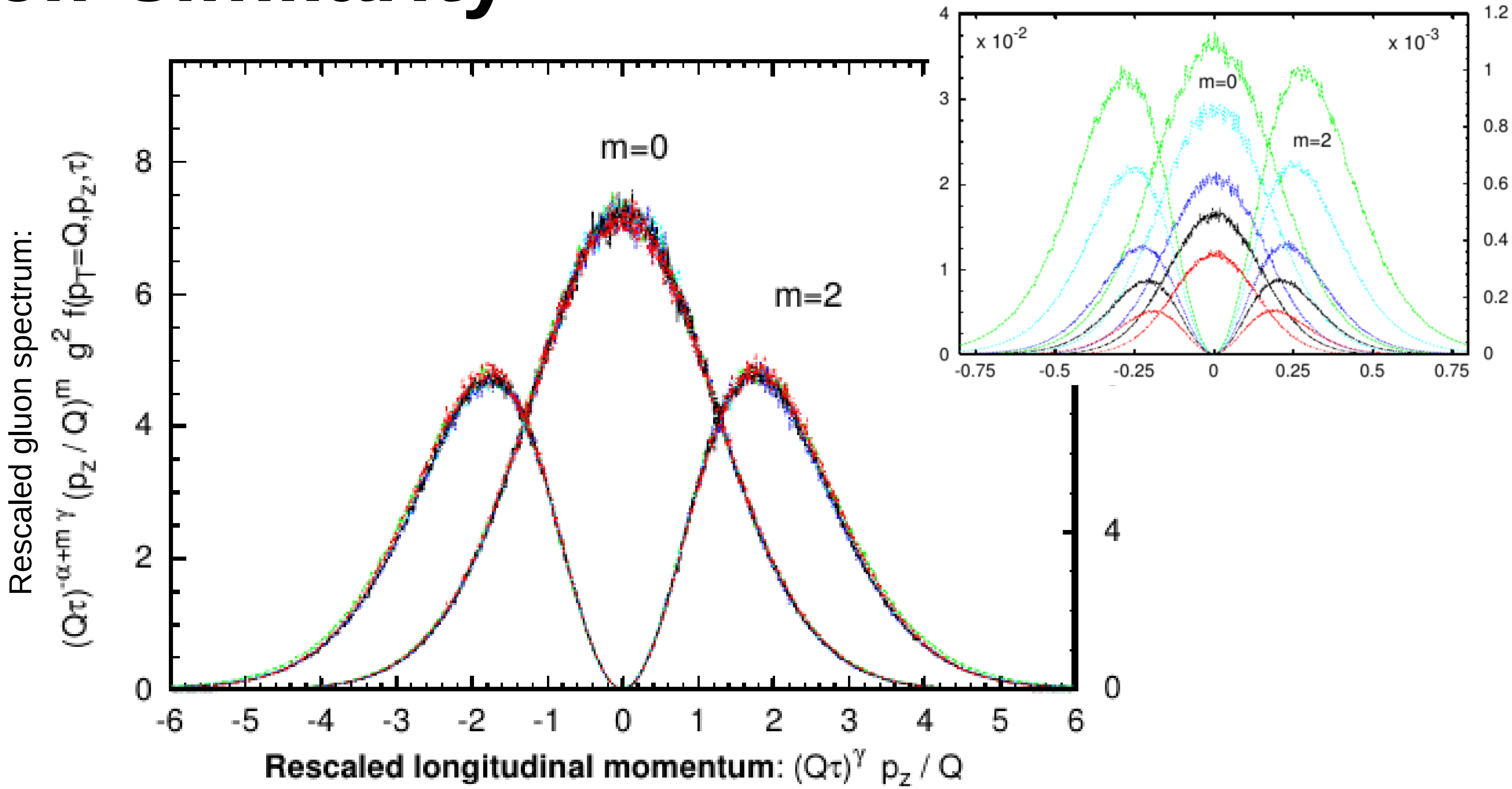
Longitudinal spectrum



However not strong enough to compensate completely for the red shift due to the longitudinal expansion.

(Berges, Boguslavski, SS, Venugopalan PRD 89 074011 & arXiv:1311.3005)

Self-similarity



The system reaches a classical turbulent attractor, where the space-time evolution becomes self-similar, i.e. $f(p_T, p_Z, \tau) = (Q\tau)^{\alpha} f_S((Q\tau)^{\beta} p_T, (Q\tau)^{\gamma} p_Z)$

with scaling exponents $\alpha \approx -2/3$, $\beta \approx 0$, $\gamma \approx 1/3$

(Berges, Boguslavski, SS, Venugopalan PRD 89 074011 & arXiv:1311.3005)

Kinetic interpretation

Consider the Boltzmann equation

$$\left[\partial_\tau - \frac{p_Z}{\tau}\partial_{p_Z}\right] f(p_T, p_Z, \tau) = C[f](p_T, p_Z, \tau)$$

with a self-similar evolution

$$f(p_T, p_Z, \tau) = (Q\tau)^{\alpha} f_S\left((Q\tau)^{\beta} p_T, (Q\tau)^{\gamma} p_Z\right)$$

→ ***Non-thermal fixed point solution*** $(f \gg 1)$

$$\left[\alpha + \beta\, p_T \partial_{p_T} + (\gamma - 1)\, p_Z \partial_{p_Z}\right] f_S(p_T, p_Z) = Q^{-1} C[f_S](p_T, p_Z)$$

→ ***Scaling exponents determined by scaling relations for***

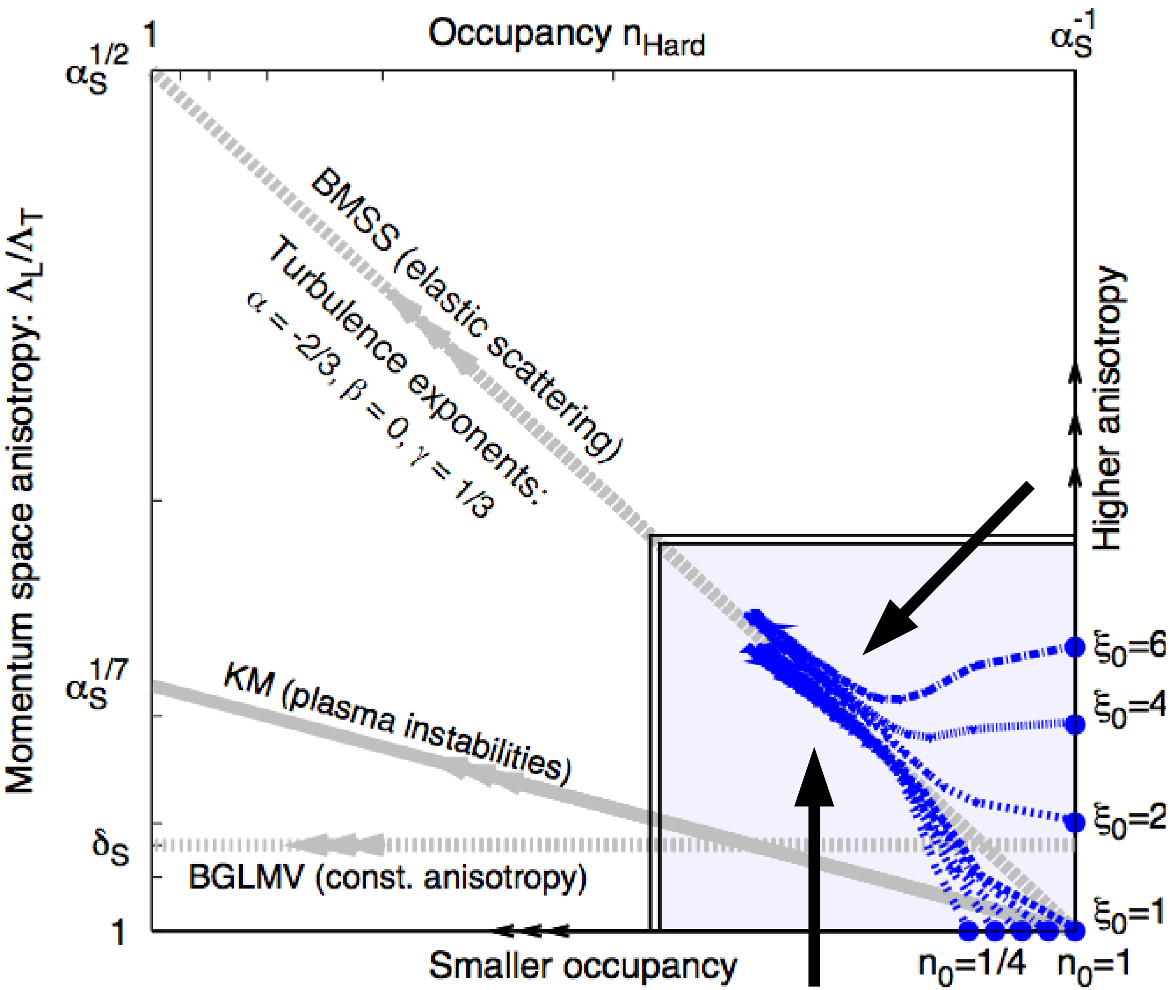
- Small angle elastic scattering $(2\alpha - 2\beta + \gamma = -1)$
- Energy conservation $(\alpha - 3\beta - \gamma = -1)$
- Particle number conservation $(\alpha - 2\beta - \gamma = -1)$

→ $\alpha = -2/3, \beta = 0, \gamma = 1/3$ ***in excellent agreement with lattice data!***

Confirms "bottom-up" thermalization scenario (Baier et al. PLB 502 (2001) 51-58***)***

(Berges, Boguslavski, SS, Venugopalan PRD 89 074011 & arXiv:1311.3005)

The attractor solution

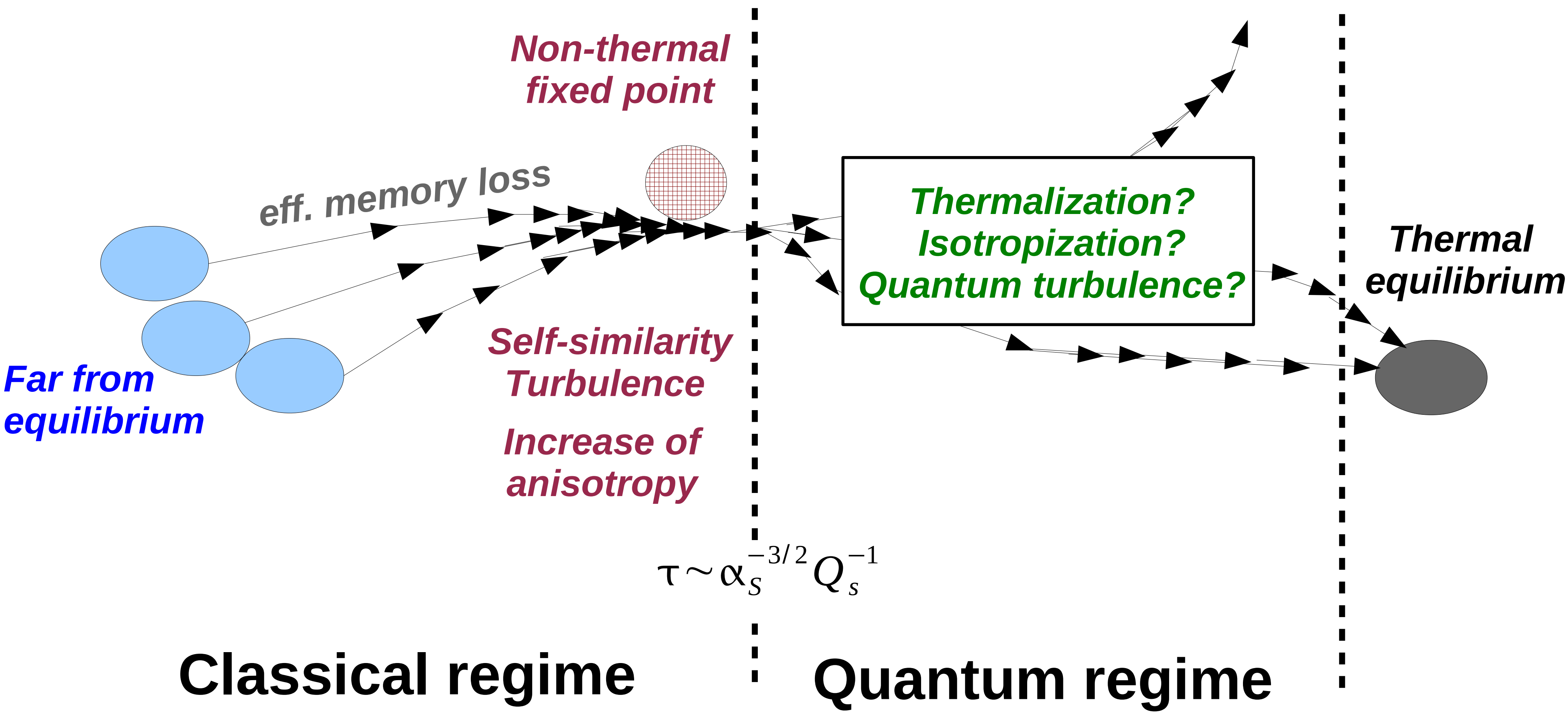


- Universal scaling behavior for different initial conditions
- Qualitative agreement with the first stage of the "bottom-up" thermalization scenario (Baier et al. PLB 502 (2001) 51-58)

→ No sign that plasma instabilities play a significant role

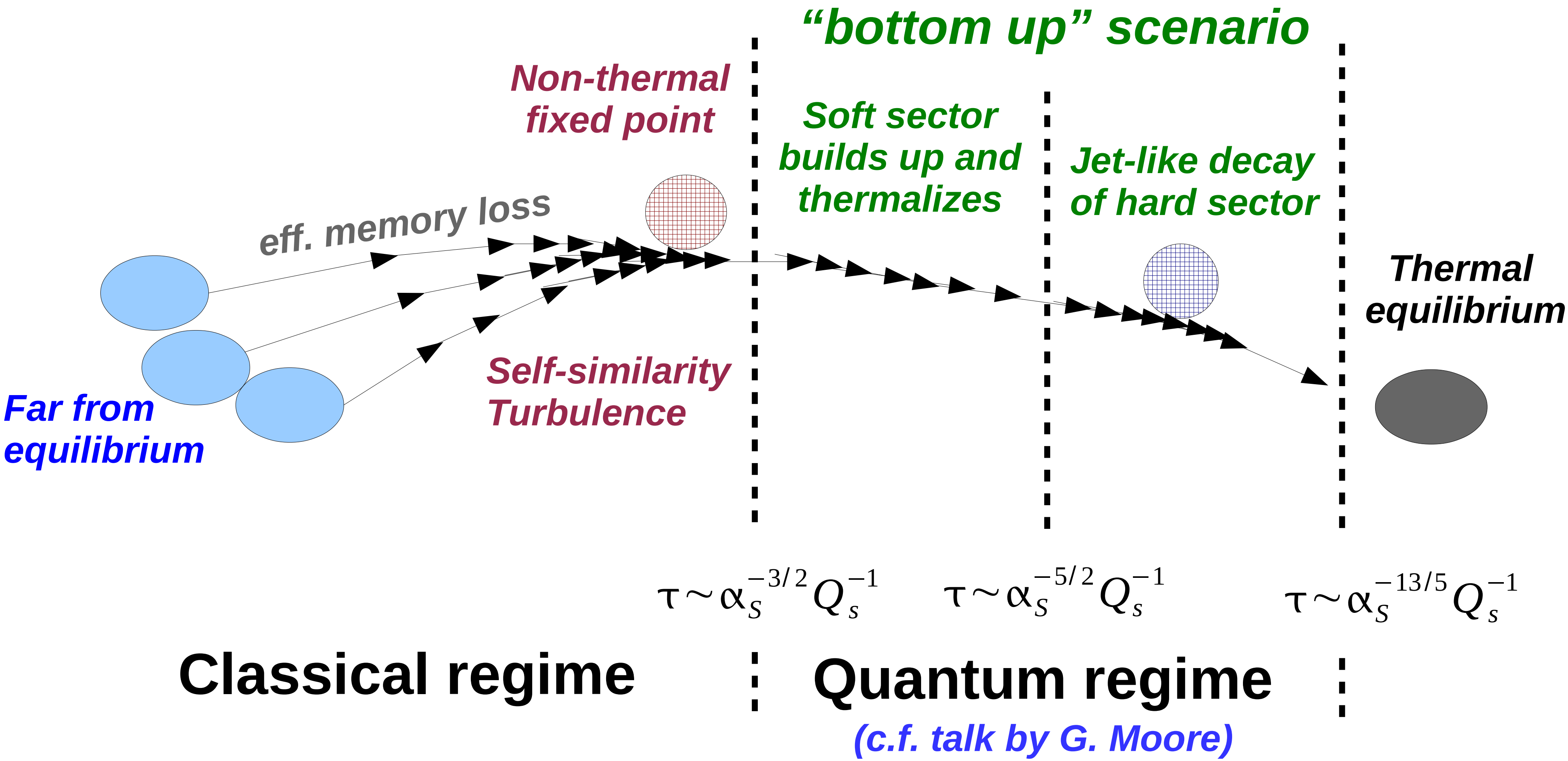
Thermalization process

The expanding plasma exhibits a ***self-similar evolution.*** However, at the end of the classical regime the system is ***still far from equilibrium***



Thermalization process

Classical statistical simulations no longer applicable in the quantum regime. However kinetic theory predictions provide route to thermal equilibrium



Conclusion & Outlook

- ***Classical-statistical lattice simulations*** can be used to study the non-equilibrium dynamics from first principles in weak coupling limit.

- Within the common range of validity lattice simulations agree well with kinetic theory *(c.f. talk by G. Moore)*

- Turbulent ***thermalization process*** appears as a generic feature of strongly correlated many-body systems across different energy scales *('big bang', 'little bang', 'ultracold bang')*

Open questions:

- How is the weak-coupling attractor approached for themost realistic initial conditions?

- How exactly is isotropization/thermalization achieved in the quantum regime? *(c.f. Kurkela, Lu arXiv:1405.6318)*

- Can we reliably perform simulations directly at larger values of the coupling? *(c.f. Epelbaum, Gelis PRL 111 (2013) 232301; BBSV arXiv:1312.5216)*

- How to compare weak coupling and strong coupling results?