Holographic thermalization at strong and intermediate coupling

Aleksi Vuorinen

Mainz Institute for Theoretical Physics, 8.8.2014

R. Baier, S. Stricker, O. Taanila, AV, 1205.2998 (JHEP), 1207.1116 (PRD)
D. Steineder, S. Stricker, AV, 1209.0291 (PRL), 1304.3404 (JHEP)
S. Stricker, 1307.2736 (EPJ-C)
V. Keränen, H. Nishimura, S. Stricker, O. Taanila and AV, 1405.7015

UNIVERSITY OF HELSINKI

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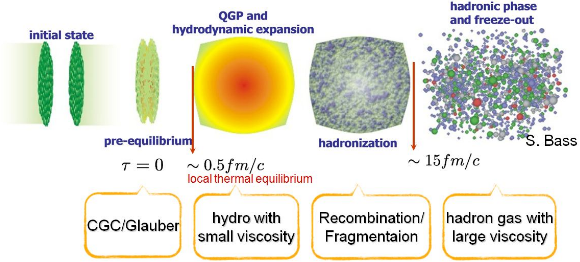
Describing a heavy ion collision

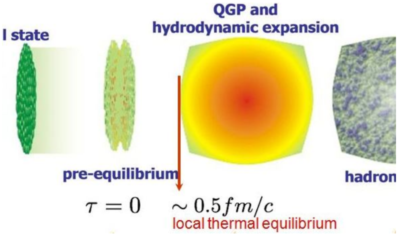


Nontrivial observation: Hydrodynamic description of fireball evolution extremely successful with few theory inputs

1. Relatively easy: Equation of state and freeze-out criterion
2. Hard: Transport coefficients of the plasma (η, ζ, ...)
3. Very hard: Initial conditions & onset time τ_{hydro}

Surprise from RHIC/LHC: Extremely fast equilibration into almost ‘ideal fluid’ behavior — hard to explain via weakly coupled quasiparticle picture

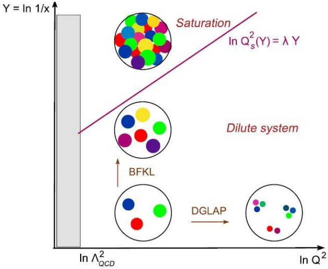
Thermalization puzzle

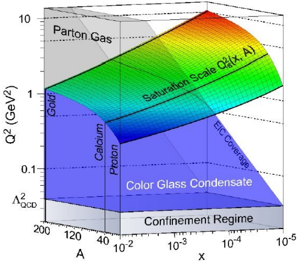


Major challenge for theorists: Understand the fast dynamics that take the system from complicated, far-from-equilibrium initial state to near-thermal ‘hydrodynamized’ plasma

Characteristic energy scales and nature of the plasma evolve fast (running coupling) $\Rightarrow$ Need to efficiently combine both perturbative and nonperturbative machinery

Initial state of a heavy ion collision





D. Triantafyllopoulos

At RHIC/LHC energies, initial state typically characterized by

- Existence of one hard scale: Saturation momentum $Q_s \gg \Lambda_{\text{QCD}}$
- Overoccupation of gluons: $f(q < Q_s) \sim 1/\alpha_s$
- High anisotropy: $q_z \ll q_\perp$

Early dynamics of a high energy collision

When describing early (initially perturbative) dynamics of a collision, need to take into account

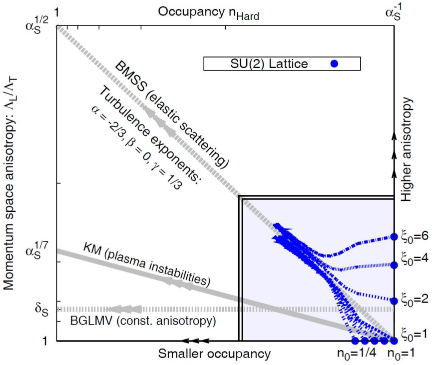
- Longitudinal expansion of the system
- Elastic and inelastic scatterings
- Plasma instabilities

Traditional field theory tools available:

1. Classical (bosonic) lattice simulations — work as long as occupation numbers large[1] (quantum time evolution not feasible)
2. Weak coupling expansions; disagreement related to the role of plasma instabilities, affecting α_s scaling of τ_{therm}[2]
3. Effective kinetic theory — works at smaller occupancies, but breaks down in the description of IR physics[3]

[1] Berges et al., 1303.5650, 1311.3005
[2] Baier et al., hep-ph/0009237; Kurkela, Moore, 1107.5050; Blaizot et al., 1107.5296
[3] Abraao York, Kurkela, Lu, Moore, 1401.3751

Thermalization in a weakly coupled plasma



Inelastic scatterings drive bottom-up thermalization

- Soft modes quickly create thermal bath
- Hard splittings lead to $q \sim Q_s$ particles being eaten by the bath

Numerical evolution of expanding SU(2) YM plasma seen to always lead to Baier-Mueller-Schiff-Son type scaling at late times (Berges et al., 1303.5650, 1311.3005)

Ongoing debate about the role of instabilities in hard interactions, argued to lead to slightly faster thermalization: $\tau_{\mathrm{KM}} \sim \alpha_s^{-5/2}$ vs. $\tau_{\mathrm{BMSS}} \sim \alpha_s^{-13/5}$

Thermalization beyond weak coupling

Remarkable progress for the early weak-coupling dynamics of a high energy collision. However, extension of the results to realistic heavy ion collision problematic:

- System clearly not asymptotically weakly coupled $\Rightarrow$ Direct use of perturbative results requires bold extrapolation
- Dynamics classical only in an overoccupied system — works only for the early dynamics of the system
- Kinetic theory description misses important physics, e.g. instabilities

In absence of nonperturbative first principles techniques, clearly room for alternative approaches

- Needed in particular: Tool to address dynamical problems in strongly coupled field theory — interesting problem in itself!

The holographic way

All approaches to (thermal) QCD are some types of *systematically improvable* approximations: pQCD, lattice QCD, effective theories, ...

Why not consider a different expansion point: SU(N_c) gauge theory with

- N_c taken to infinity
- Large 't Hooft coupling $\lambda = g^2 N_c$
- Additional adjoint fermions and scalars to make the theory $\mathcal{N} = 4$ supersymmetric and conformal

AdS/CFT conjecture (Maldacena, 1997):

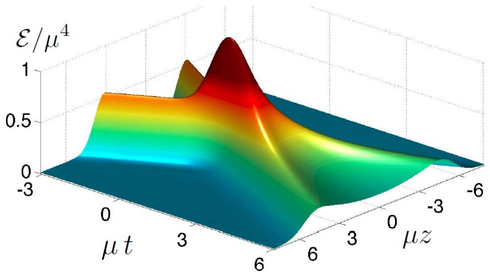
- IIB string theory in $AdS_5 \times S_5$ exactly dual to $\mathcal{N} = 4$ Super Yang-Mills (SYM) theory living on the 4d Minkowskian boundary of the AdS space
- Strongly coupled, $N_c \to \infty$ SYM $\leftrightarrow$ Classical supergravity

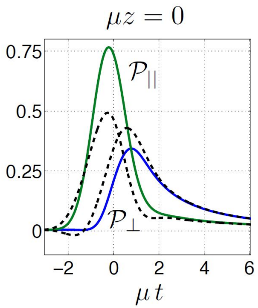
Strong coupling thermalization

Due to conformality, SYM theory very different from QCD at $T = 0$. However:

- At finite temperature, systems much more similar
  - Both describe deconfined plasmas with Debye screening, finite static correlation length,...
  - Conformality and SUSY broken due to introduction of T
- Most of the above limits systematically improvable
- *Very* nontrivial field theory problems mapped to classical gravity

Strong coupling thermalization





Chesler, Yaffe, 1011.3562

Important lessons from gauge/gravity calculations at infinite coupling:

- Thermalization always of top-down type (causal argument)
- Thermalization time naturally short, $\sim 1/T$
- Hydrodynamization $\neq$ Thermalization, isotropization

Bridging the gap

Obviously, it would be valuable to bring the two limiting cases closer to each other — and to a realistic setting. Is it possible to:

- Extend weak coupling picture to lower energies, with $\alpha_s(Q) \sim 1$?
- Marry weak coupling description of the early dynamics with strong coupling evolution?
- Bring field theory used in gauge/gravity calculations closer to real QCD?
    - Finite coupling & N_c, dynamical breaking of conformal invariance,...

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Rest of the talk: Attempt to relax the $\lambda = \infty$ (and conformality) approximation in studies of holographic thermalization

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AdS/CFT duality: $T = 0$

- Original conjecture: SU(N_c) $\mathcal{N} = 4$ SYM in $\mathbb{R}^{1,3} \leftrightarrow$ IIB ST in AdS$_5 \times$S$_5$

"center" of AdS — $r = 0$

boundary — $r = \infty$

- Pure AdS metric corresponds to vacuum state of the CFT

$$ds^2 = L^2\left(- r^2 dt^2 + \frac{dr^2}{r^2} + r^2 d\mathbf{x}^2 \right)$$

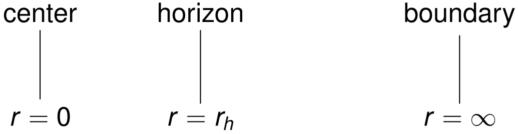
- Dictionary: CFT operators $\leftrightarrow$ bulk fields, with identification

$$(L/l_s)^4 = \lambda, \quad g_s = \lambda/(4\pi N_c)$$

$\Rightarrow$ Strongly coupled, large-N_c QFT $\leftrightarrow$ Classical sugra

AdS/CFT duality: $T \neq 0$

- Strongly coupled large-N_c SYM plasma in thermal equilibrium $\leftrightarrow$ Classical gravity in AdS black hole background



- Metric now features event horizon at $r = r_h$ ($L \equiv 1$ from now on)

$$ds^2 = -r^2(1 - r_h^4/r^4)dt^2 + \frac{dr^2}{r^2(1 - r_h^4/r^4)} + r^2 d\mathbf{x}^2$$

- Identification of field theory temperature with Hawking temperature of the black hole $\Rightarrow T = r_h/\pi$

AdS/CFT duality: Thermalizing system

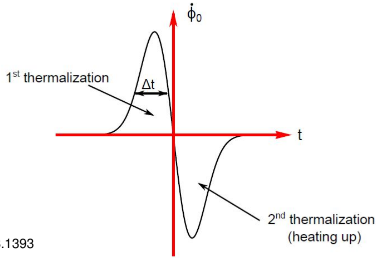
- Simplest way to take system out of equilibrium: Radial gravitational collapse of a thin shell (Danielsson, Keski-Vakkuri, Kruczenski)

center	horizon	shell	boundary
$r = 0$	$r = r_h$	$r = r_s$	$r = \infty$

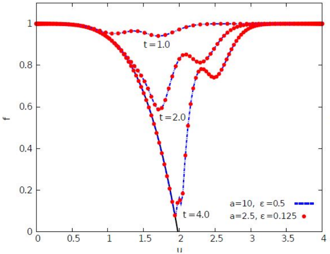
- Metric defined in a piecewise manner:

$$ds^2 = -r^2 f(r) dt^2 + \frac{dr^2}{r^2 f(r)} + r^2 d\mathbf{x}^2, \quad f(r) = \begin{cases} f_-(r) \equiv 1\,, & \text{for } r < r_s \\ f_+(r) \equiv 1 - \frac{r_h^4}{r^4}\,, & \text{for } r > r_s \end{cases}$$

- Shell fills entire three-space $\Rightarrow$ Translational and rotational invariance
- Field theory side: Rapid, spatially homogenous injection of energy at all scales



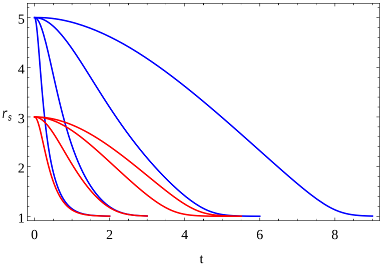
Bin Wu, 1208.1393

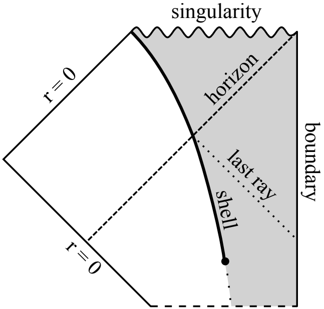


Shell can be realized by briefly turning on a spatially homogenous scalar source in the CFT, coupled to

- A marginal composite operator in the CFT
- The bulk metric through Einstein equations involving the corresponding bulk field

$$ds^2 = \frac{1}{u^2}\left(-f(u,t)\, e^{-2\delta(u,t)} dt^2 + 1/f(u,t)\, du^2 + d\mathbf{x}^2 \right), \quad u = r_h^2/r^2$$





Alternatively can send off shell from rest at finite radius r_0

- For shell EoS $p = c\varepsilon$ radical slowing down of collapse as $c \to 1/3$, assuming mass of final black hole fixed
- r_0 only hard scale in the problem $\Rightarrow$ Tempting to speculate about relation to the saturation momentum

Holographic Green's functions

In- and off-equilibrium correlators offer useful tool for studying thermalization:

- Poles of retarded thermal Green's functions give dispersion relation of field excitations: Quasiparticle / quasinormal mode spectrum
- Time dependent off-equilibrium Green's functions probe how fast different energy (length) scales equilibrate
- Related to measurable quantities, e.g. particle production rates

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Example 1: EM current correlator $\langle J_\mu^{\rm EM} J_\nu^{\rm EM} \rangle$ — photon production

- Obtain by adding to the SYM theory a U(1) vector field coupled to a conserved current corresponding to a subgroup of $SU(4)_R$
- Excellent phenomenological probe of thermalization because of photons' weak coupling to plasma constituents

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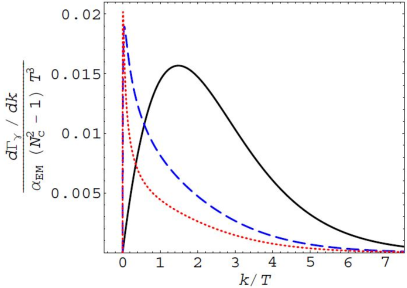

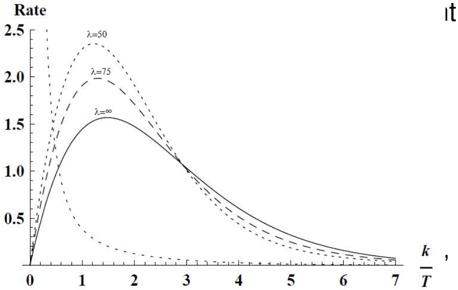

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Example 2: Energy momentum tensor correlators $\langle T_{\mu\nu} T_{\alpha\beta} \rangle$ related to e.g. shear and bulk viscosities and dual to metric fluctuations $h_{\mu\nu}$

- Scalar channel: h_{xy}
- Shear channel: h_{tx}, h_{zx}
- Sound channel: h_{tt}, h_{tz}, h_{zz}, h_{ii}

Recipe for the retarded correlator

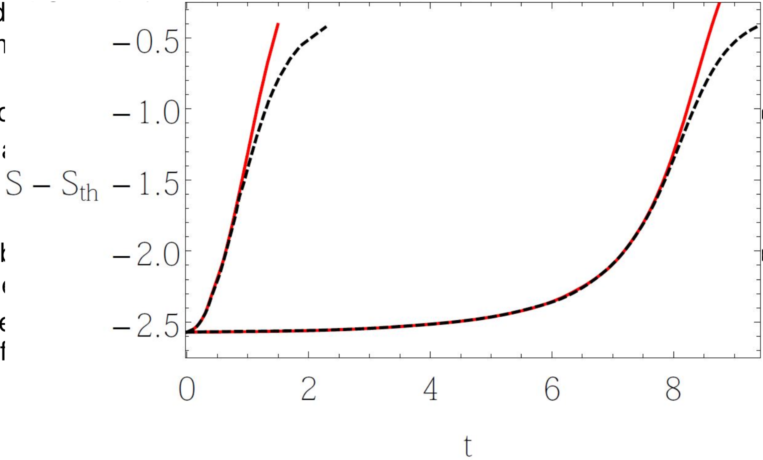
Retarded Green's functions obtainable within the *quasistatic approximation* with small modifications to the original Son-Starinets recipe:

1. Solve classical EoM for the relevant bulk field inside and outside the shell
2. Match solutions at the shell using Israel junction conditions
   - Quasistatic limit: Ignore time derivatives
   - With $r_s > r_h$, the outside solution has also an outgoing component
3. Obtain the Green's function from the behavior of the outside solution near the boundary
4. Repeat steps 1-3 for different values of r_s/r_h; if desired, combine this information with time-dependence from shell's trajectory
   - Conformal EoS $\Rightarrow$ Parametrically slower evolution

Recipe for the retarded correlator

Retard ... *tion*
with sn

1. Sc ... e shell
2. Ma



3. Ob ... on near
   the
4. Re ... is
   inf

Beyond infinite coupling: α' corrections

Recall key relation from AdS/CFT dictionary: $(L/l_s)^4 = L^4/\alpha'^2 = \lambda$, with α' the inverse string tension

- To go beyond $\lambda = \infty$ limit, need to add α' terms to the sugra action, i.e. determine the first non-trivial terms in a small-curvature expansion
- Leading order corrections $\mathcal{O}(\alpha'^3) = \mathcal{O}(\lambda^{-3/2})$

End up dealing with $\mathcal{O}(\alpha'^3)$ improved type IIB sugra

$$\begin{aligned}
S_{IIB} &= \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G}\left(R_{10} - \frac{1}{2}(\partial\phi)^2 - \frac{F_5^2}{4\cdot 5!} + \gamma e^{-\frac{3}{2}\phi}(C+\mathcal{T})^4\right), \\
\mathcal{T}_{abcdef} &\equiv i\nabla_a F^+_{bcdef} + \frac{1}{16}\left(F^+_{abcmn}F^+_{def}{}^{mn} - 3F^+_{abfmn}F^+_{dec}{}^{mn}\right), \\
F^+ &\equiv \frac{1}{2}(1+*)F_5, \quad \gamma \equiv \frac{1}{8}\zeta(3)\lambda^{-3/2}
\end{aligned}$$

$\Rightarrow \gamma$-corrected metric and EoMs for different fields

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Quasinormal mode spectra at finite coupling

Analytic structure of retarded thermal Green's functions $\Rightarrow$ Dispersion relation of field excitations

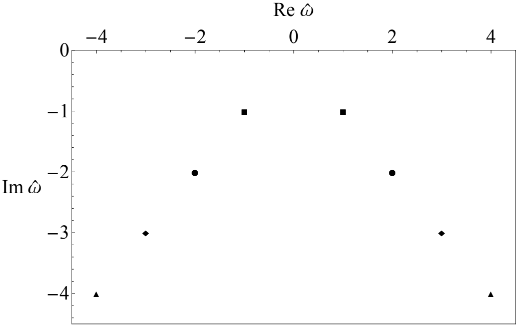
$$\omega_n(k) = E_n(k) + i\Gamma_n(k)$$

Striking difference between weakly and strongly coupled systems:

- At weak coupling *long-lived quasiparticles* with $\Gamma_n \ll E_n$
- At strong coupling *quasinormal mode spectrum*

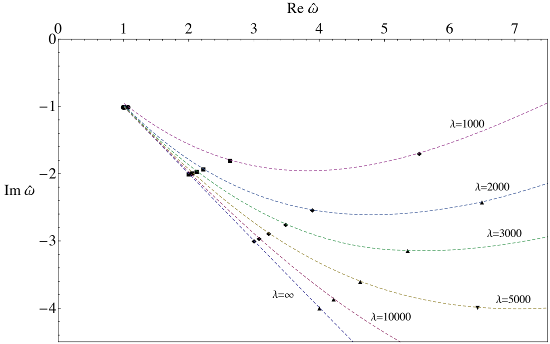
$$\hat{\omega}_n|_{k=0} = \frac{\omega_n|_{k=0}}{2\pi T} = n\,(\pm 1 - i)$$

QNMs at infinite coupling: Photons



Pole structure of EM current correlator displays usual quasinormal mode spectrum at $\lambda = \infty$. How about at finite coupling?

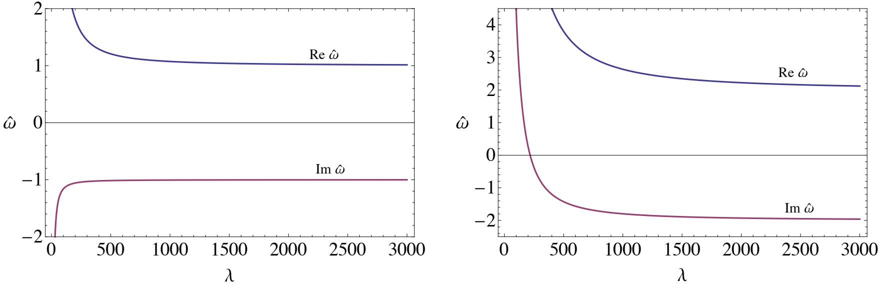
QNMs at finite coupling: Photons



Effect of decreasing λ: Widths of the excitations consistently decrease $\Rightarrow$ Modes become longer-lived

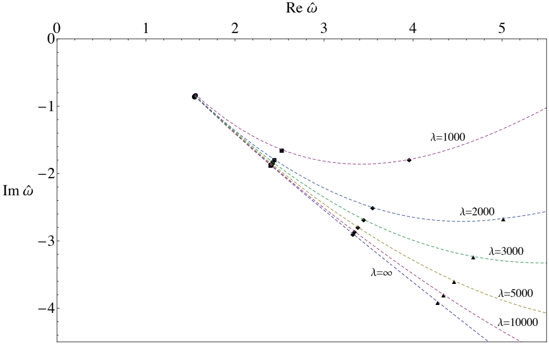
NB: Convergence of strong coupling expansion not guaranteed, when $\hat{\omega}_n|_{k=0} = n(\pm 1 - i) + \xi_n/\lambda^{3/2}$ shifted from $\lambda = \infty$ value by $\mathcal{O}(1)$ amount

QNMs at finite coupling: Photons



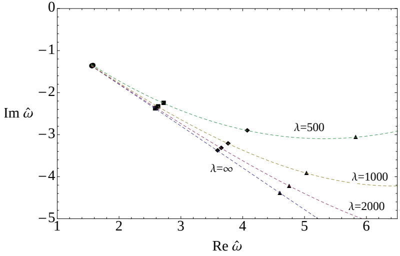
Zoom-in to the two lowest modes, $n = 1$ and 2: Sensitivity to γ-corrections grows rapidly with n

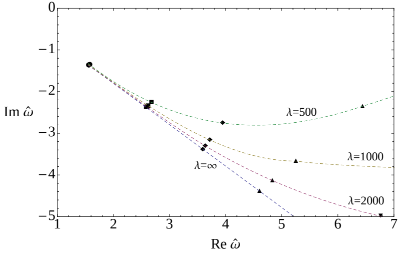
QNMs at finite coupling: Photons



Similar shift at nonzero three-momentum: $k = 2\pi T$

QNMs at finite coupling: $T_{\mu\nu}$ correlators





Same effect also in the shear (left) and sound (right) channels of energy-momentum tensor correlators (here $k = 0$)

Outside the $\lambda = \infty$ limit, the response of a strongly coupled plasma to infinitesimal perturbations appears to change, with the QNM spectrum moving towards the real axis

What happens if we take the system further away from equilibrium?

Off-equilibrium Green's functions: Definitions

Natural quantities to study: Spectral density $\chi(\omega, k) \equiv \mathrm{Im}\,\Pi_{\mathrm{R}}(\omega, k)$ and related particle production rate (here photons)

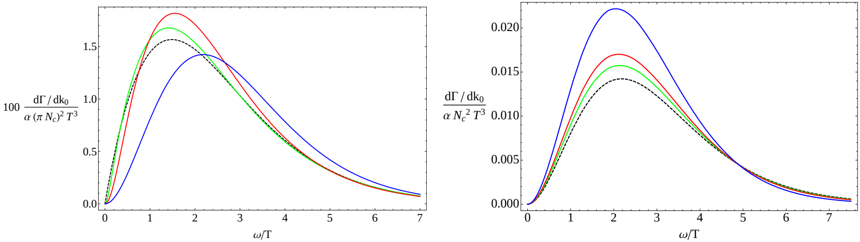
$$k^0 \frac{d\Gamma_\gamma}{d^3k} = \frac{1}{4\pi k}\frac{d\Gamma_\gamma}{dk_0} = \frac{\alpha_{\mathrm{EM}}}{4\pi^2}\eta^{\mu\nu}\Pi^{<}_{\mu\nu}(k_0 \equiv \omega, k) = \frac{\alpha_{\mathrm{EM}}}{4\pi^2}\eta^{\mu\nu} n_B(\omega)\chi^{\mu}_{\mu}(\omega, k)$$

Useful measure of 'out-of-equilibriumness': Relative deviation of spectral density from the thermal limit

$$R(\omega, k) \equiv \frac{\chi(\omega, k) - \chi_{\mathrm{therm}}(\omega, k)}{\chi_{\mathrm{therm}}(\omega, k)}$$

Important consistency check: $R \to 0$, as $r_s \to r_h$

Production rates: Real (on-shell) photons

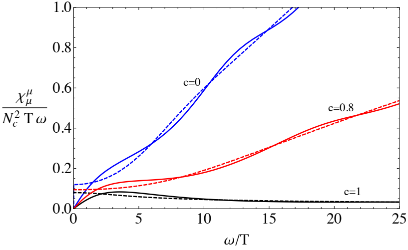


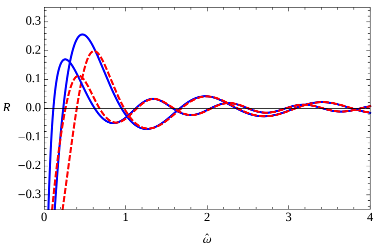
Left: Photon production rate for $\lambda = \infty$ and $r_s/r_h = 1.1, 1.01, 1.001, 1$

Right: Photon production rate for $r_s/r_h = 1.01$ and $\lambda = \infty$, 120, 80, 40

Note the much weaker dependence on λ than in the QNM spectrum

Spectral density and R at $\lambda = \infty$: Photons



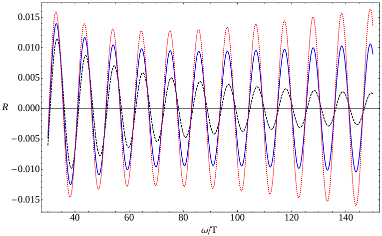


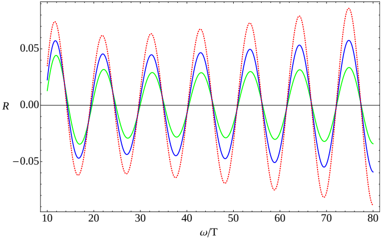
Left: Photon spectral functions for different virtualities ($c = k/\omega$) in thermal equilibrium and $r_s/r_h = 1.1$

Right: Relative deviation $R \equiv (\chi - \chi_{\rm th})/\chi_{\rm th}$ for dileptons ($c = 0$) with $r_s/r_h = 1.1$ and 1.01 together with analytic WKB results, valid at large ω

Note: Clear top-down thermalization pattern (as always at $\lambda = \infty$)

Relative deviation at finite λ: Photons

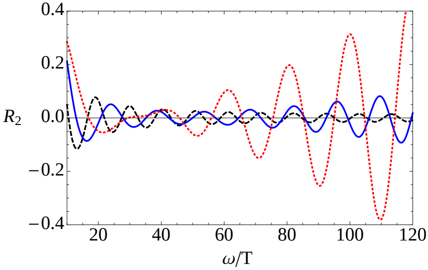


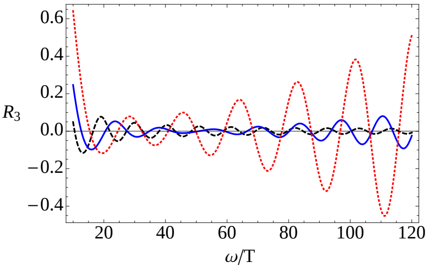


Relative deviation $R \equiv (\chi - \chi_{\text{th}})/\chi_{\text{th}}$ for on-shell photons with $r_s/r_h = 1.01$ and $\lambda = \infty$, 500, 300 (left) and 150, 100, 75 (right)

NB: Change of pattern with decreasing λ: UV modes no longer first to thermalize.

Relative deviation at finite λ: $T_{\mu\nu}$ correlators





Relative deviation $R \equiv (\chi - \chi_{\text{th}})/\chi_{\text{th}}$ in the shear and sound channels for $r_s/r_h = 1.2$, $\lambda = 100$, and $k/\omega = 0$ (black), 6/9 (blue) and 8/9 (red)

Reliability of results

So what to make of all this? Indications of the holographic plasma starting to behave like a system of weakly coupled quasiparticles, or simply

- ... due to the breakdown of some approximation?
  - Quasistatic limit OK as long as $\omega/T \gg 1$
  - Strong coupling expansion applied with care: (NLO-LO)/LO $\lesssim \mathcal{O}(1/10)$
- ... a peculiarity of the channels considered?
  - EM current and $T_{\mu\nu}$ correlators probe system in different ways
  - Recent results for purely geometric probes display different behavior[4]
- ... a sign of the unphysical nature of the collapsing shell model?
  - Difficult to rule out. However, at least QNM results universal.

$\therefore$ Clearly, more work needed to generalize results — in particular to more realistic and dynamical models of thermalization

[4] Galante, Schvellinger, 1205.1548

Implications for holography

For a given quantity,

$$X(\lambda) \quad = \quad X(\lambda = \infty) \times \left(1 + X_1/\lambda^{3/2} + \mathcal{O}(1/\lambda^3)\right)$$

define critical coupling λ_c such that $|X_1/\lambda_c^{3/2}| = 1$. Then:

Quantity	$\lambda_\mathbf{c}$
Pressure	0.9
Transport/hydro coeffs. (η/s, τ_H, κ)	7 ± 1
Spectral densities in equilibrium	$\lambda_c(\omega = 0) = 40$, $\lambda_c(\omega \to \infty) = 0.8$, ...
Quasinormal mode n for photons / $T_{\mu\nu}$	$\lambda_c(n = 1) = 200$, $\lambda_c(n = 2) = 500$ $\lambda_c(n = 3) = 1000$,...

Lesson: What is weak/strong coupling depends strongly on the quantity.
Thermalization appears to be sensitive to strong coupling corrections.

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Take home messages

1. Holographic (thermalization) calculations can — and should — be taken away from $\lambda = \infty$ limit
2. QNM spectrum and thermalization related properties particularly sensitive to strong coupling corrections: At $\lambda \sim 10$ no longer within strong coupling regime
3. Tentative indications that a holographic system obtains weakly coupled characteristics within the realm of a strong coupling expansion
   - QNM poles flow in the direction of a quasiparticle spectrum
   - Top-down thermalization pattern weakens and shifts towards bottom-up