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Effective Field Theory Approach to Flavour and Dark Matter

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Outline:

- Introduction: Higher Dimensional Operators
- Lepton Flavour Violation
  - Effects of gauge invariant dim-6 operators
- Determination of the V_{ub} and V_{cb}
  - Can NP explain the differences in the determinations
- Dark Matter detection
  - SI cross section
  - Dim 5
  - Dim 6
  - Dim 7
- Conclusions

Introduction

Higher dimensional operators

- NP at the scale Λ>v must be invariant under the SM gauge group
- The heavy degrees of freedom can be integrated out T. Appelquist, J. Carazzone

⇒ The resulting effective operators must be Lorentz invariant, respect the SM gauge group and are suppressed by powers of 1/Λ.

B. Grzadkowski et al., arXiv:1008.4884
W. Buchmüller, D. Wyler, Nucl.Phys. B268 (1986) 621-653

Why EFT methods:

- Argue indecently of the specific NP model
- Properly connect physics at different scales via
  - Running
  - Mixing
  - Matching
- Correlate different experiments (complementarily of searches)
- Can be easily extended to account for DM, right-handed neutrinos, …

Operator classification

$$L_{SM} = L_{SM}^{(4)} + 1/\Lambda \sum_k C_k^{(5)} Q_k^{(5)} + 1/\Lambda^2 \sum_k C_k^{(6)} Q_k^{(6)} + O\left(1/\Lambda^3\right)$$

- Dim 5: 1 operator, the Weinberg operator

$$Q_{\nu\nu} = \varepsilon_{jk}\varepsilon_{mn}\varphi^j\varphi^m \left(\ell_p^k\right)^T C l_r^n = \left(\varphi^\dagger l_p\right)^T C\left(\varphi^\dagger \ell_r\right)$$

- Dim 6: 59 operators
  - 30 four-fermion operators $Q_{le} = \left(\bar{\ell}_p \gamma_\mu \ell_r\right)\left(\bar{e}_s \gamma^\mu e_t\right)$
  - 4 pure field-strength tensor operators $Q_G = f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
  - 3 Higgs operators $Q_\varphi = (\varphi\varphi^\dagger)^3$
  - 8 Higgs-field-strength operators $Q_{\varphi G} = \varphi^\dagger \varphi G_{\mu\nu}^A G^{A\mu\nu}$
  - 3 Higgs-fermion operators $Q_{\varphi e} = \varphi^\dagger \varphi \bar{\ell}_i \varphi e_j$
  - 8 "magnetic" operators $Q_{eB} = \bar{\ell}_i \sigma_{\mu\nu} e_j \varphi B^{\mu\nu}$
  - 8 Higgs-fermion-derivative $Q_{\varphi\ell}^{(1)} = \varphi^\dagger D_\mu \varphi \bar{\ell}_i \gamma^\mu \ell_j$

General procedure

- Perform EW symmetry breaking
- Derive the Feynman rules
- Calculate the Feynman diagrams
- Perform the matching (integrate out W,Z, t and h)
- RGE evolution to the low scale
- Calculation of decay width, cross sections, etc.

Lepton Flavour Violation with dim-6 operators

A.C., S. Najjari, J. Rosiek, arXiv:1312.0632
A.C., M. Hoferichter, M. Procura, arXiv:1404.7134

Lepton flavour violation

In the SM (with massive neutrinos) lepton flavour violations is extremely suppressed by the small neutrino masses: O(10^{-52})

⇒ Any observation of LFV would establish physics beyond the SM.

- Current best limits on $\mu \to e$ transitions (from PSI):

$$\mathrm{Br}\left[\mu \to e\gamma\right] \leq 5.7\times10^{-13}$$

$$\mathrm{Br}\left[\mu \to eee\right] \leq 1\times10^{-12}$$

$$\mathrm{Br}_{\mathrm{Au}}^{\mathrm{conv}}\left[\mu \to e\right] \leq 7\times10^{-13}$$

- Future prospects:

$\mathrm{Br}\left[\mu \to e\gamma\right] \leq 6\times10^{-14}$ PSI

$\mathrm{Br}_{\mathrm{Al}}^{\mathrm{conv}}\left[\mu \to e\right] \leq 5\times10^{-17}$ FNAL, J-PARC

$\mathrm{Br}\left[\mu \to eee\right] \leq 7\times10^{-17}$ PSI (proposed)

Observabels

Contributions of dim-6 operators to

- $\ell \to \ell' \gamma$ (one loop)
- $\tau \to \mu\mu\mu, \tau \to e\mu\mu$, etc.
- EDM, AMM (one loop)
- $Z \to \ell\ell'$
- $\mu \to e$ conversion

At leading loop-order and at leading order in $1/\Lambda^2$ and m_ℓ / m_W

Operators for LFV

$\ell\ell\ell\ell$		$\ell\ell X\varphi$		$\ell\ell\varphi^2 D$ and $\ell\ell\varphi^3$	
$Q_{\ell\ell}$	$(\bar\ell_i\gamma_\mu\ell_j)(\bar\ell_k\gamma^\mu\ell_l)$	Q_{eW}	$(\bar\ell_o\sigma^{\mu\nu}e_j)\tau^I\varphi W^I_{\mu\nu}$	$Q^{(1)}_{\varphi\ell}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar\ell_i\gamma^\mu\ell_j)$
Q_{ee}	$(\bar e_i\gamma_\mu e_j)(\bar e_k\gamma^\mu e_l)$	Q_{eB}	$(\bar\ell_i\sigma^{\mu\nu}e_j)\varphi B_{\mu\nu}$	$Q^{(3)}_{\varphi\ell}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu\varphi)(\bar\ell_i\tau^I\gamma^\mu\ell_j)$
$Q_{\ell e}$	$(\bar\ell_i\gamma_\mu\ell_j)(\bar e_k\gamma^\mu e_l)$			$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar e_i\gamma^\mu e_j)$
				$Q_{e\varphi 3}$	$(\varphi^\dagger\varphi)(\bar\ell_i e_j\varphi)$
$\ell\ell qq$					
$Q^{(1)}_{\ell q}$	$(\bar\ell_i\gamma_\mu\ell_j)(\bar q_k\gamma^\mu q_l)$	$Q_{\ell d}$	$(\bar\ell_i\gamma_\mu\ell_j)(\bar d_k\gamma^\mu d_l)$	$Q_{\ell u}$	$(\bar\ell_i\gamma_\mu l_j)(\bar u_k\gamma^\mu u_l)$
$Q^{(3)}_{\ell q}$	$(\bar\ell_i\gamma_\mu\tau^I\ell_j)(\bar q_k\gamma^\mu\tau^I q_l)$	Q_{ed}	$(\bar e_i\gamma_\mu e_j)(\bar d_k\gamma^\mu d_l)$	Q_{eu}	$(\bar e_i\gamma_\mu e_j)(\bar u_k\gamma^\mu u_l)$
Q_{eq}	$(\bar e_i\gamma^\mu e_j)(\bar q_k\gamma_\mu q_l)$	$Q_{\ell edq}$	$(\bar\ell^a_i e_j)(\bar d_k q^a_l)$	$Q^{(1)}_{\ell equ}$	$(\bar\ell^a_i e_j)\varepsilon_{ab}(\bar q^b_k u_l)$
				$Q^{(3)}_{\ell equ}$	$(\bar\ell^a_i\sigma_{\mu\nu}e_a)\varepsilon_{ab}(\bar q^b_k\sigma^{\mu\nu}u_l)$

ℓ : left-handed lepton doublet

e: right-handed charged lepton

φ: Higgs doublet

$W^{\mu\nu}$: $SU(2)$ field-strength tensor

$B^{\mu\nu}$: $U(1)$ field-strength tensor

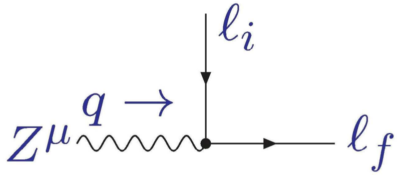
i, j, k, l : flavour indices

$\overleftrightarrow{D}_\mu$: covariant derivative

Derivation of the modified Feynman rules

- After EW symmetry breaking
- General R_X gauge

Example: Z-lepton coupling



$$i\left(\gamma^\mu\left[\Gamma_{fi}^{ZL}P_L+\Gamma_{fi}^{ZR}P_R\right]+i\sigma^{\mu\nu}\left[C_{fi}^{ZL}P_L+C_{fi}^{ZR}P_R\right]q_\nu\right)$$

$$\Gamma_{fi}^{ZL}=\frac{e}{2s_Wc_W}\left(\frac{v^2}{\Lambda^2}\left(C_{\phi\ell}^{(1)fi}+C_{\phi\ell}^{(3)fi}\right)+\left(1-2s_W^2\right)\delta_{fi}\right)$$

$$\Gamma_{fi}^{ZR}=\frac{e}{2s_Wc_W}\left(\frac{v^2}{\Lambda^2}C_{\phi e}^{fi}-2s_W^2\delta_{fi}\right)$$

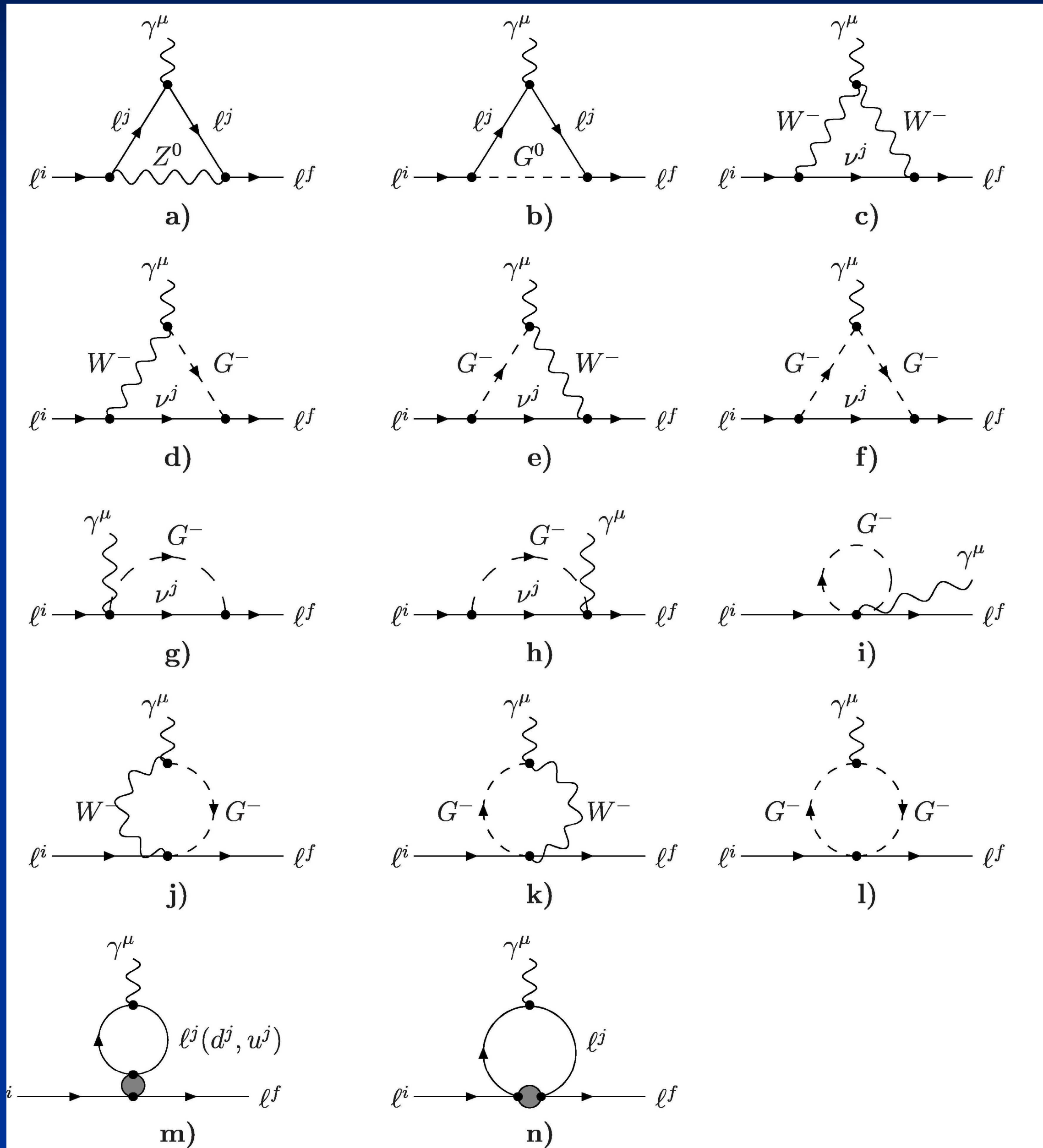
$$C_{fi}^{ZR}=C_{if}^{ZL\star}=-\frac{v\sqrt{2}}{\Lambda^2}\left(s_WC_{eB}^{fi}+c_WC_{eW}^{fi}\right)$$

Calculation of the diagrams

$\ell \to \ell' \gamma$

EDM

AMM



$$\mathrm{Br}\left[\ell_i \to \ell_f \gamma\right] = \frac{m_{\ell_i}^3}{16\pi\Lambda^4 \Gamma_{\ell_i}} \left(\left|F_{TR}^{fi}\right|^2 + \left|F_{TL}^{fi}\right|^2 \right)$$

$$F_{TL}^{ZG\,fi} = \frac{4e\left[\left(C_{\varphi\ell}^{(1)fi} + C_{\varphi l}^{(3)fi}\right) m_f (1+s_W^2) - C_{\varphi e}^{fi} m_i \left(\frac{3}{2} - s_W^2\right)\right]}{3(4\pi)^2}$$

$$F_{TR}^{ZG\,fi} = \frac{4e\left[\left(C_{\varphi\ell}^{(1)fi} + C_{\varphi l}^{(3)fi}\right) m_i (1+s_W^2) - C_{\varphi e}^{fi} m_f \left(\frac{3}{2} - s_W^2\right)\right]}{3(4\pi)^2}$$

$$F_{TL}^{WG\,fi} = -\frac{10 e m_f C_{\varphi\ell}^{(3)fi}}{3(4\pi)^2}$$

$$F_{TR}^{WG\,fi} = -\frac{10 e m_i C_{\varphi\ell}^{(3)fi}}{3(4\pi)^2}$$

Checks:

- Gauge invariant structure of the operators
- R_χ gauge

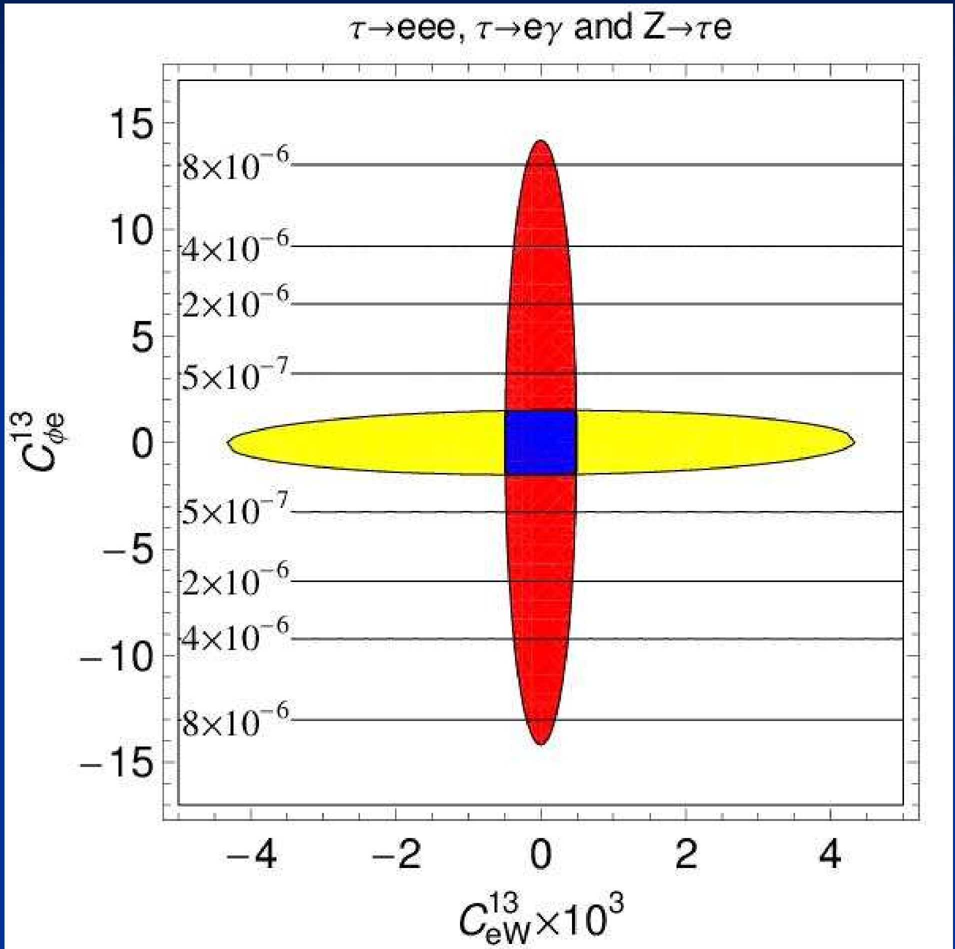
$$F_{TL}^{4\ell\,fi} = \frac{2e}{(4\pi)^2} \sum_{j=1}^3 C_{\ell e}^{fjji} m_j$$

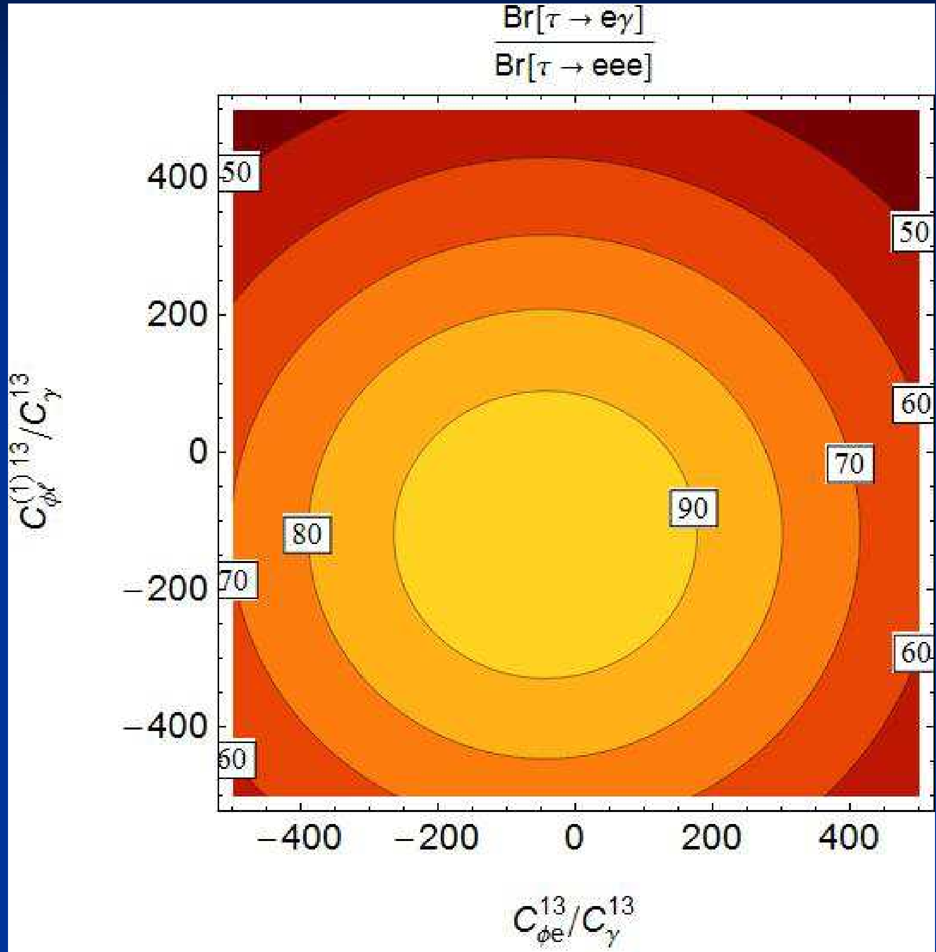
$$F_{TR}^{4\ell\,fi} = \frac{2e}{(4\pi)^2} \sum_{j=1}^3 C_{\ell e}^{jifj} m_j$$

$$F_{TL}^{ql\,fi} = -\frac{16e}{3(4\pi)^2} \sum_{j=1}^3 C_{\ell equ}^{(3)fijj\star} m_{u_j} \left(\Delta - \log\frac{m_{u_j}^2}{\mu^2}\right)$$

$$F_{TR}^{ql\,fi} = -\frac{16e}{3(4\pi)^2} \sum_{j=1}^3 C_{\ell equ}^{(3)fijj} m_{u_j} \left(\Delta - \log\frac{m_{u_j}^2}{\mu^2}\right)$$

Numerical results



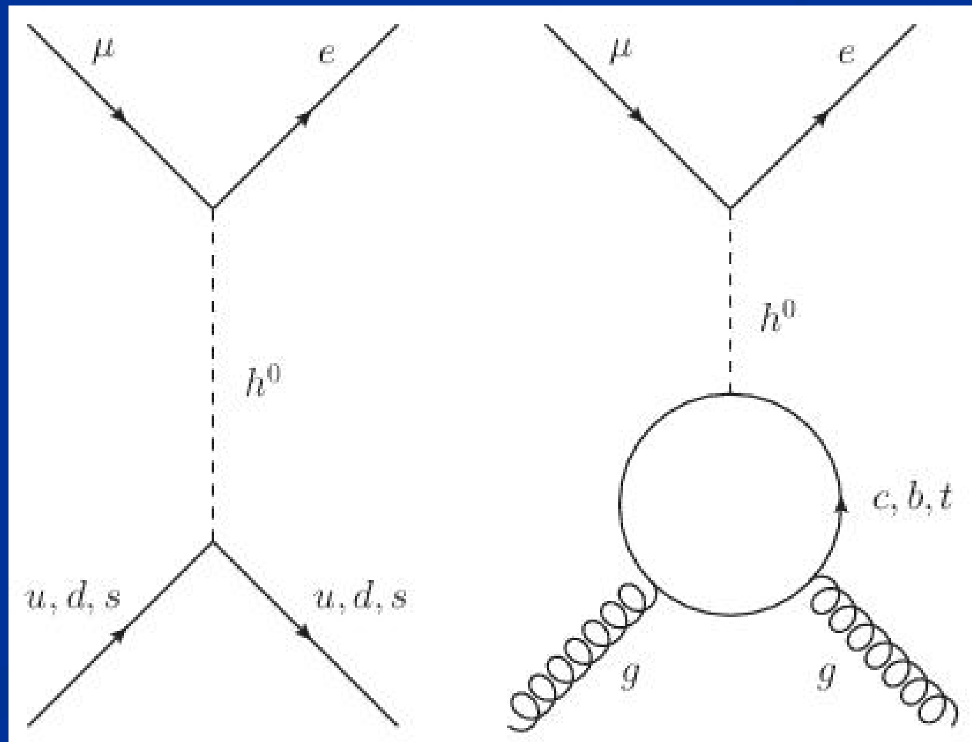


$\tau \to eee$ (yellow)

$\tau \to e\gamma$ (red)

$\mu \to e$ conversion and and Higgs mediated flavour violation

- Higgs contributions to $\mu \to e\gamma$ and $\mu \to eee$ by small Yukawa couplings
- Contributions to $\mu \to e$ conversion involve also heavy quarks



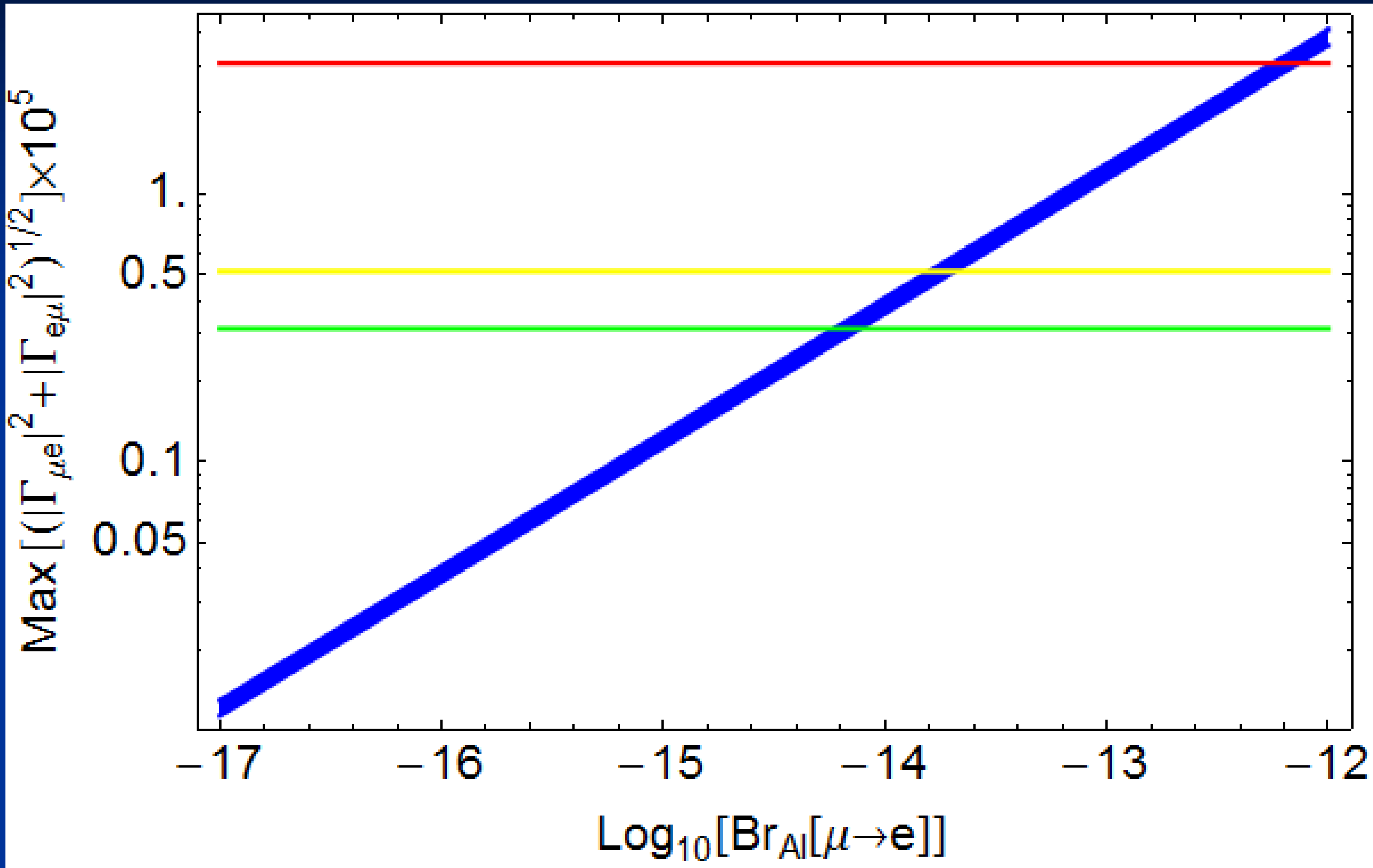
⇒ $\mu \to e$ conversion sensitive to Higgs mediated FV

dim-6 operator

$$\mathcal{O}_{e\phi}^{ij} = \left(\phi^\dagger \phi\right)\left(\overline{\ell}_i e_j \phi\right)$$

$$\Gamma_{\ell_f \ell_i}^{h^0} = -\frac{m_{\ell_i}}{v} + \frac{1}{\sqrt{2}} \frac{v^2}{\Lambda^2} \tilde{C}_{e\phi}^{fi}, \quad \tilde{C}_{e\phi}^{fi} = \left(U_\ell^{L\dagger} C_{e\phi} U_\ell^R\right)_{fi}$$

$\mu \to e$ conversion



$\mu \to e$ conversion

$\mu \to e\gamma$ (upgrade 1)

$\mu \to e\gamma$ (now)

$\mu \to e\gamma$ (upgrade 2)

$\Gamma_{\mu e}$: μ-e-h^0 coupling

Determination of V_{ub} and V_{cb}

A.C., Stefan Pokorski, arXiv:1407.1320

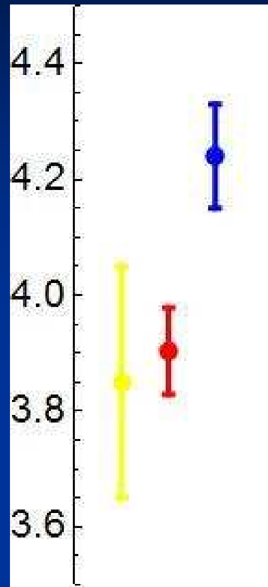
Different determinations

- V_{cb}

$$|V_{cb}| = (4.242 \pm 0.086) \times 10^{-2} \quad \text{(inclusive)}$$

$$|V_{cb}| = (3.904 \pm 0.075) \times 10^{-2} \quad (B \to D^* \ell \nu)$$

$$|V_{cb}| = (3.850 \pm 0.191) \times 10^{-2} \quad (B \to D \ell \nu)$$



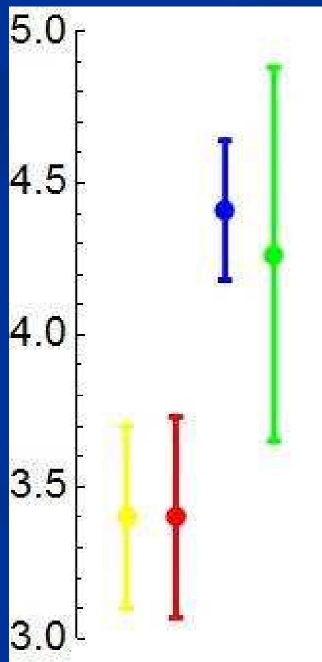
- V_{ub}

$$|V_{ub}| = (4.41^{+0.21}_{-0.23}) \times 10^{-3} \quad \text{(inclusive)}$$

$$|V_{ub}| = (3.40^{+0.38}_{-0.33}) \times 10^{-3} \quad (B \to \pi \ell \nu)$$

$$|V_{ub}| = (4.3 \pm 0.6) \times 10^{-3} \quad (B \to \tau \nu)$$

$$|V_{ub}| = (3.4 \pm 0.3) \times 10^{-3} \quad (B \to \rho \ell \nu)$$



Effective operators (at the B scale)

B. Dassinger et al. arXiv:0803.3561, S. Faller et al. arXiv:1105.3679

- Four-fermion operators

$$O_R^S = \bar{\ell} P_L \nu \bar{q} P_R b$$
$$O_L^S = \bar{\ell} P_L \nu \bar{q} P_L b$$
$$O_L^T = \bar{\ell} \sigma_{\mu\nu} P_L \nu \bar{q} \sigma^{\mu\nu} P_L b$$

contribute

$\sim |C_L^T|^2$ all decays

$\sim |C_R^S + C_L^S|^2 \quad B \to D(\pi)\ell\nu$

$\sim |C_R^S - C_L^S|^2 \quad B \to D^*(\rho)\ell\nu$

$\sim |C_R^S|^2 + |C_L^S|^2$ inclusive

➡ Cannot explain the differences in the determinations

- Modified W coupling

$$H_{eff} = \frac{4G_F V_{qb}}{\sqrt{2}} \bar{\ell} \gamma^\mu P_L \nu \left((1 + c_L^{qb}) \bar{q} \gamma_\mu P_L b + g_L^{qb} \bar{q} i \overleftrightarrow{D}_\mu P_L b + d_L^{qb} i\partial^\nu \left(\bar{q} i \sigma_{\mu\nu} P_L b \right) + L \to R \right)$$

Effects of NP

Exclusive determination at zero recoil:

- V_{cb}

$$V_{cb} = \frac{V_{cb}^{\mathrm{SM}}}{1 + c_L^{cb} + c_R^{cb} - 1.6\mathrm{GeV}(d_R^{cb} + d_L^{cb}) + 5.5\mathrm{GeV}(g_R^{cb} + g_L^{cb})} \quad (B \to D\ell\nu)$$

$$V_{cb} = \frac{V_{cb}^{\mathrm{SM}}}{1 + c_L^{cb} - c_R^{cb} + 3.3\mathrm{GeV}(d_R^{cb} - d_L^{cb})} \quad (B \to D^*\ell\nu)$$

- V_{ub}

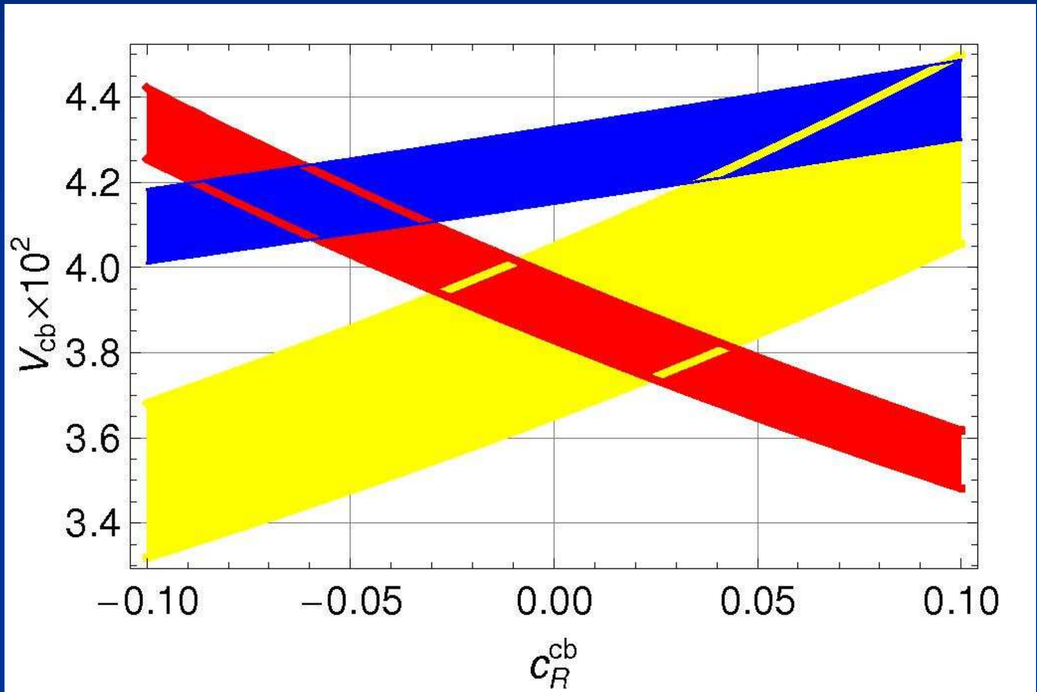
$$V_{ub} = \frac{V_{ub}^{\mathrm{SM}}}{1 + c_L^{ub} + c_R^{ub} - 4.9\mathrm{GeV}(d_R^{ub} + d_L^{ub}) + 5.5\mathrm{GeV}(g_R^{ub} + g_L^{ub})} \quad (B \to \pi\ell\nu)$$

$$V_{ub} = \frac{V_{ub}^{\mathrm{SM}}}{1 + c_L^{ub} - c_R^{ub} + 4.5\mathrm{GeV}(d_R^{ub} - d_L^{ub})} \quad (B \to \rho\ell\nu)$$

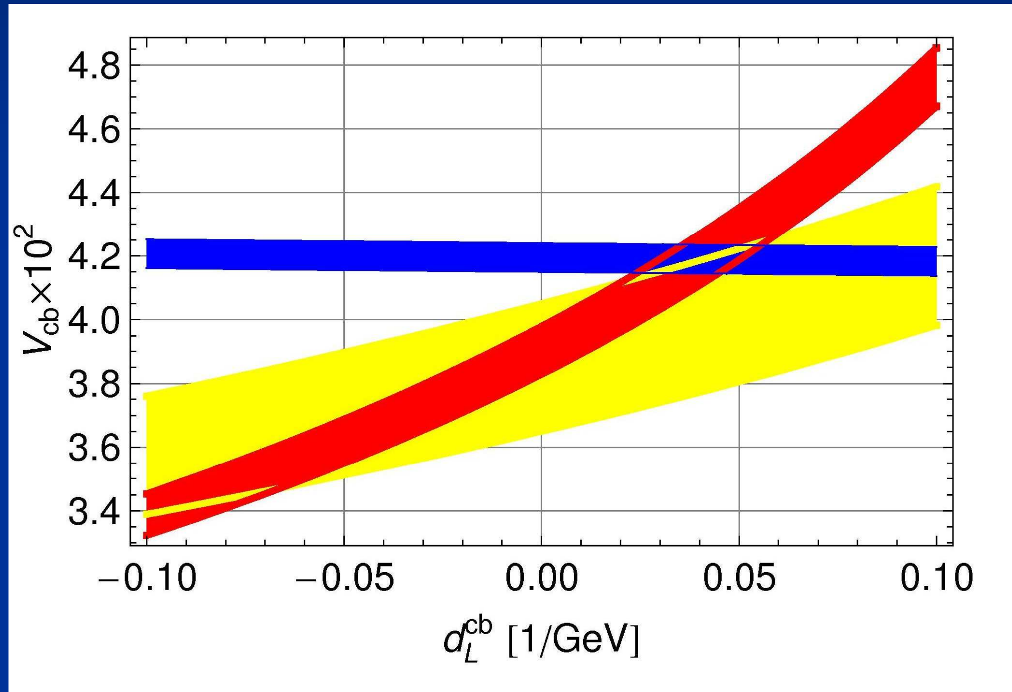
Inclusive determination on weakly affected

New Physics Effects in V_{cb}

Right-handed W coupling



„magnetic“ operator



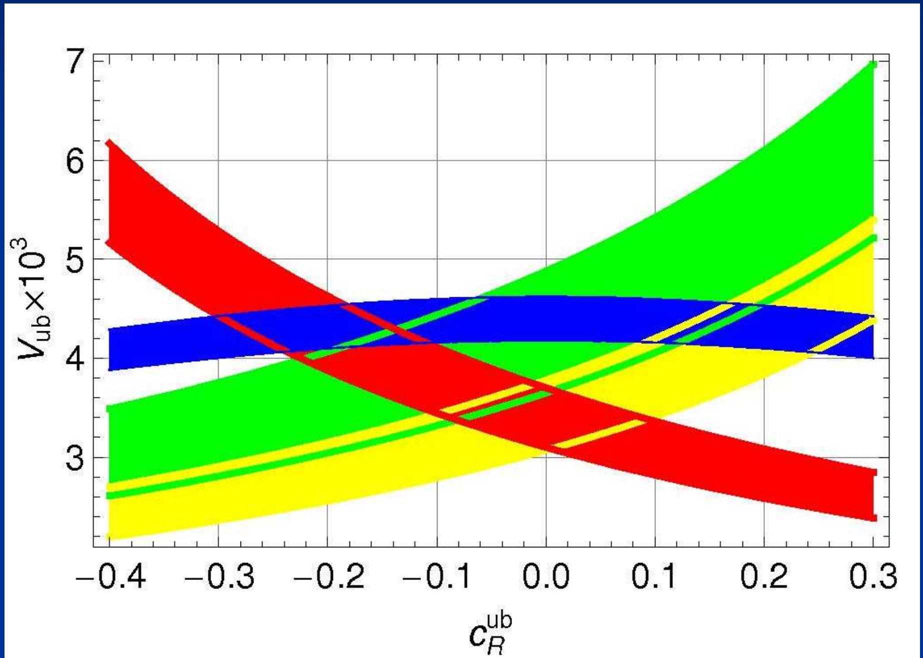
inclusive

$B \to D^* \ell \nu$

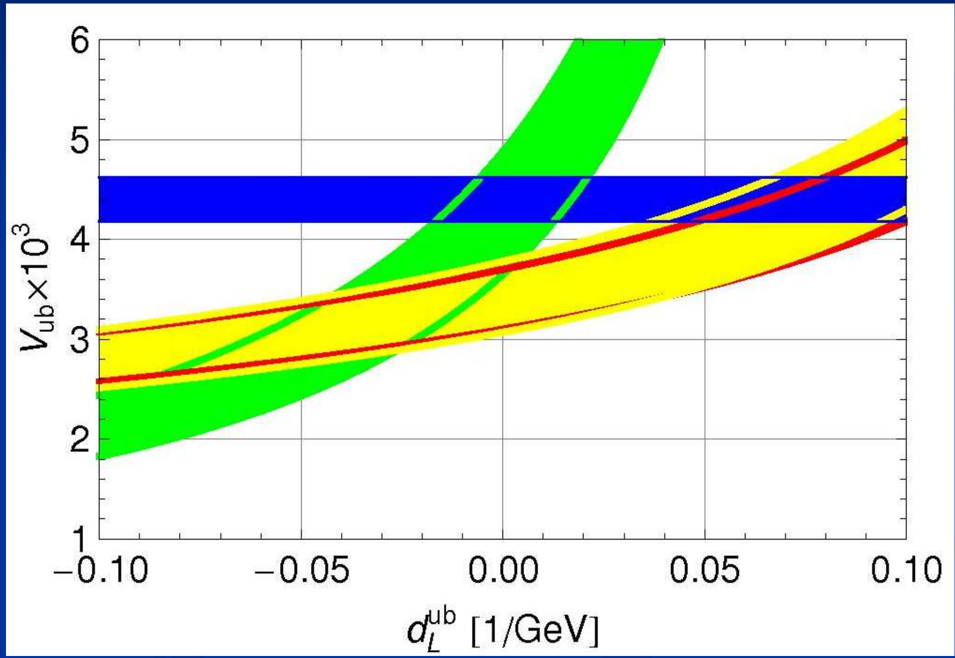
$B \to D \ell \nu$

New Physics Effects in V_{ub}

Right-handed W coupling



„magnetic“ operator



inclusive

$B \to \pi \ell \nu$

$B \to \tau \nu$

$B \to \rho \ell \nu$

Results

- In terms of SU(2) invariant operators d_L corresponds to

$$Q_{uW}^{ij} = 1/\Lambda^2 \left(\bar{q}_i \sigma^{\mu\nu} u_j\right) \tau^I \tilde{\varphi} W_{\mu\nu}^I$$

- Direct connection to Z-quark couplings
- Excluded order one corrections to Z-bb couplings

NP at the scale Λ cannot explain the differences in the determinations of V_{ub} and V_{cb}.

Effective field theory approach to Dark Matter

A.C., F. d'Eramo, M. Procura, arXiv:1402.1173
A.C., M. Hoferichter, M. Procura arXiv:1312.4951
A.C., U. Haisch, arXiv:1408:xxxx

Spin independent scattering cross section

- Up to Dim 7 (at the direct detection scale)

$$\sigma_N^{\mathrm{SI}} \approx \frac{m_N^2}{\pi\Lambda^4}\left|\sum_{q=u,d} C_{qq}^{VV} f_{V_q}^N + \frac{m_N}{\Lambda}\left(\sum_{q=u,d,s} C_{qq}^{SS} f_q^N - 12\pi C_{gg}^S f_Q^N\right)\right|^2$$

$$L_{eff} = \sum_X C_X O_X$$

$$O_{gg}^S = \frac{\alpha_s}{\Lambda^3}\bar{\chi}\chi G_{\mu\nu}G^{\mu\nu}$$

$$O_{qq}^{SS} = \frac{m_q}{\Lambda^3}\bar{\chi}\chi\bar{q}q$$

$$O_{qq}^{VV} = \frac{1}{\Lambda^2}\bar{\chi}\gamma^\mu\chi\bar{q}\gamma_\mu q$$

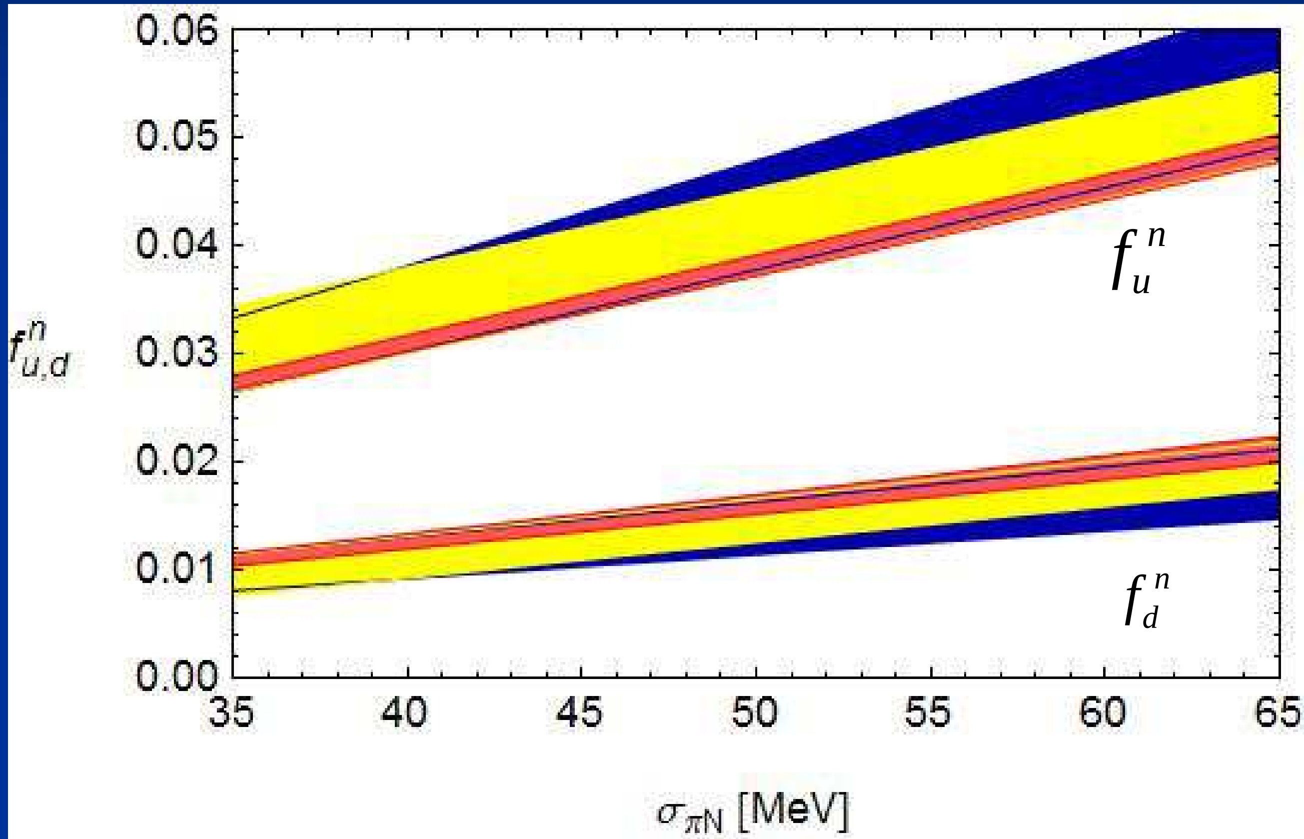
f^N : nucleon couplings

m_N : nucleon mass

The Wilson coefficients C_X must be connected to UV physics

Scalar quark content of the nucleon

- Tradiational approach: SU(3) chiral pertubation
- Better: SU(2) chiral pertubation theory and f_s from lattice



our result

SU(3)

SU(3) with f_s

EFT for Dark Matter

- We assume that DM is:
  - A SM singlet (other choices are also possible)
  - A Dirac fermion (biggest number of operators)
- Interactions of DM with the SM arise through messengers at a high scale Λ
- Construct operators which are invariant under the SM gauge group
- This scale Λ must be connected to the direct detection scale via running, mixing and threshold effects.

Operators dim-5

$$O_M^T = \frac{1}{\Lambda}\bar{\chi}\sigma^{\mu\nu}\chi B_{\mu\nu}, \quad O_{HH}^S = \frac{1}{\Lambda}\bar{\chi}\chi H^\dagger H, \quad O_{HH}^P = \frac{1}{\Lambda}\bar{\chi}\gamma^5\chi H^\dagger H$$

- O_M^T : Tree-level contribution to direct detection
- O_{HH}^P : Affects only spin dependent direct detection
- O_{HH}^S : Enters only via matching corrections

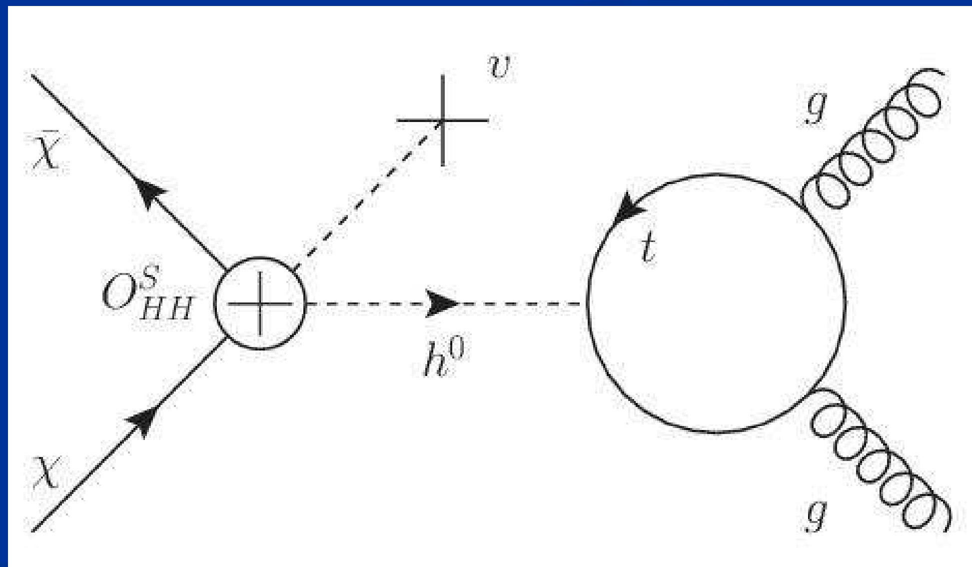
Matching: $$C_{gg}^S = \frac{1}{12\pi}\frac{\Lambda^2}{m_{h^0}^2}C_{HH}^S$$

$$C_{qq}^{SS} = -\frac{\Lambda^2}{m_{h^0}^2}C_{HH}^S$$

- Mixing turns out to be small

$$C_{qq}^{SS}(\mu_0) = \left[\frac{1}{12\pi}\left(U_{m_b,m_t}^{(5)} + 2U_{\mu_0,m_b}^{(4)}\right) - 1\right]\frac{\Lambda^2}{m_{h^0}^2}C_{HH}^S$$

$$U_{\mu,\Lambda}^{(n_f)} = \frac{-3C_F}{\pi\beta_0}\ln\frac{\alpha_s(\Lambda)}{\alpha_s(\mu)}.$$

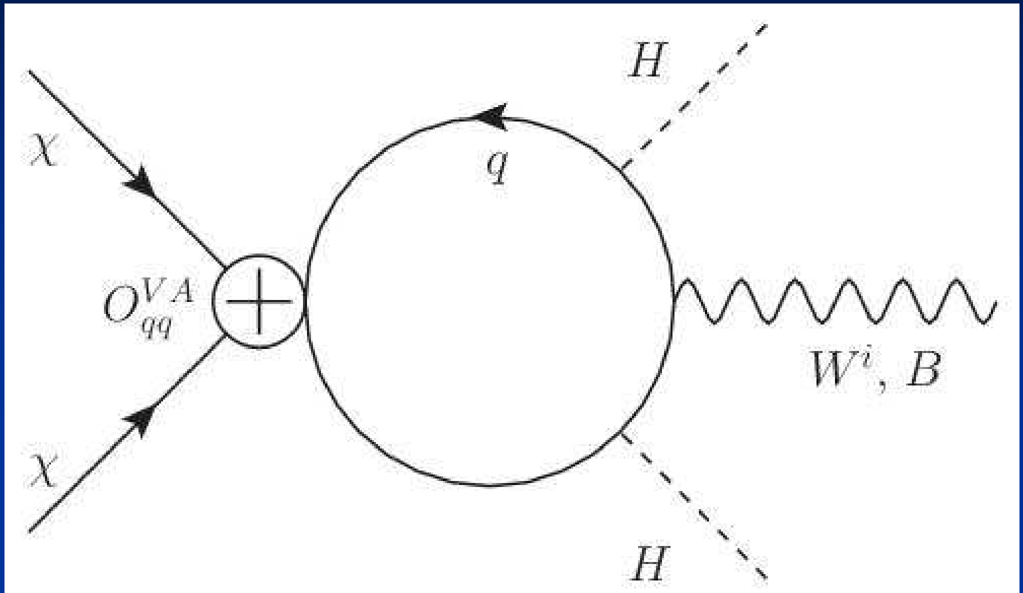


Operators dim-6

$$O_{qq}^{VV} = \frac{1}{\Lambda^2}\bar{\chi}\gamma^\mu \chi\, \bar{q}\gamma_\mu q$$

$$O_{qq}^{VA} = \frac{1}{\Lambda^2}\bar{\chi}\gamma^\mu \chi\, \bar{q}\gamma_\mu q$$

$$O_{\phi\phi D}^{V} = \frac{i}{\Lambda^2}\bar{\chi}\gamma^\mu \chi \phi^\dagger \overleftrightarrow{D}_\mu \phi$$



- No QCD effects
- EW-mixing of O_{qq}^{VA} into O_{HHD}^{V}

$$C_{\phi\phi D}^{V}(\mu) = C_{\phi\phi D}^{V}(\Lambda) - \frac{\alpha_t N_c}{\pi} C_{tt}^{VA}(\Lambda)\ln\frac{\mu}{\Lambda} - (t \to b)$$

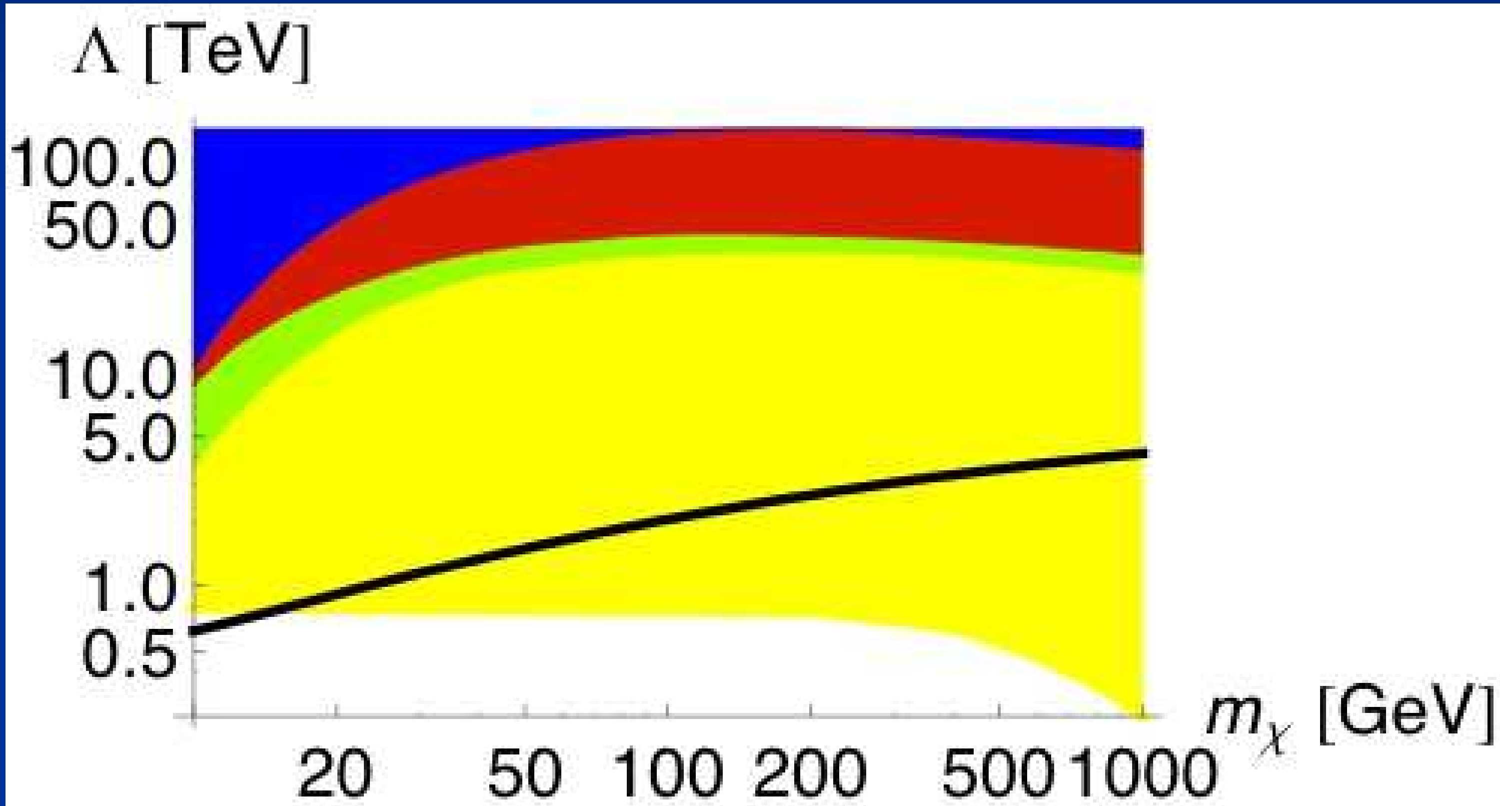
- Matching contributions

$$C_{uu}^{VV} \to C_{uu}^{VV} + \frac{1}{2}C_{HHD}^{V},\ C_{dd}^{VV} \to C_{dd}^{VV} - \frac{1}{2}C_{HHD}^{V}$$

Bounds on previously unconstrained operators

Experimental constraints

$$C_{qq}^{VA} = 1$$



XENON1T
superCDMS
LUX
LHC
relic density

Operators dim-7

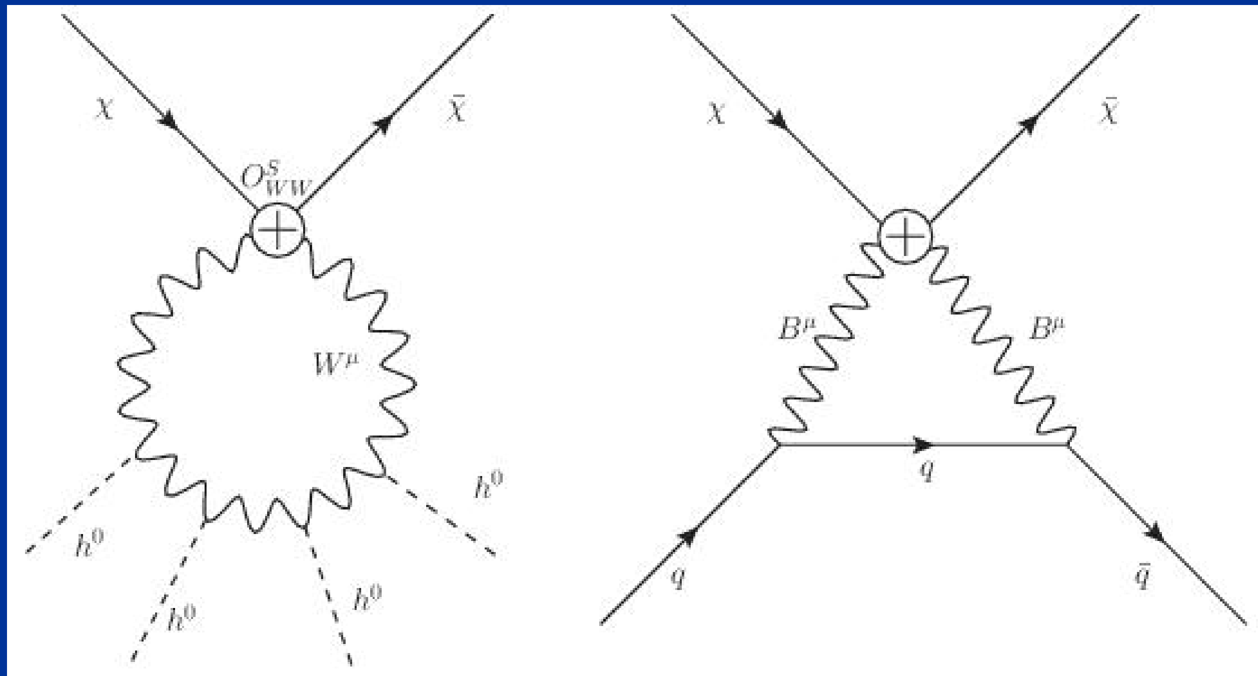
- Field strength tensors especially interesting

$$O_{BB}^{S} = \frac{1}{\Lambda^2}\bar{\chi}\chi B^{\mu\nu}B_{\mu\nu}\ , \quad O_{WW}^{S} = \frac{1}{\Lambda^2}\bar{\chi}\chi W^{\mu\nu}W_{\mu\nu}$$

- Mixing into

$$O_{4\phi}^{S} = \frac{1}{\Lambda^3}\bar{\chi}\chi\phi\phi^\dagger\phi\phi^\dagger$$

$$O_{qq}^{\phi\, SS} = \frac{Y^q}{\Lambda^3}\bar{\chi}\chi\bar{q}\phi q$$



Contributions to direct detection after
EW symmetry breaking and integrating out the Higgs.

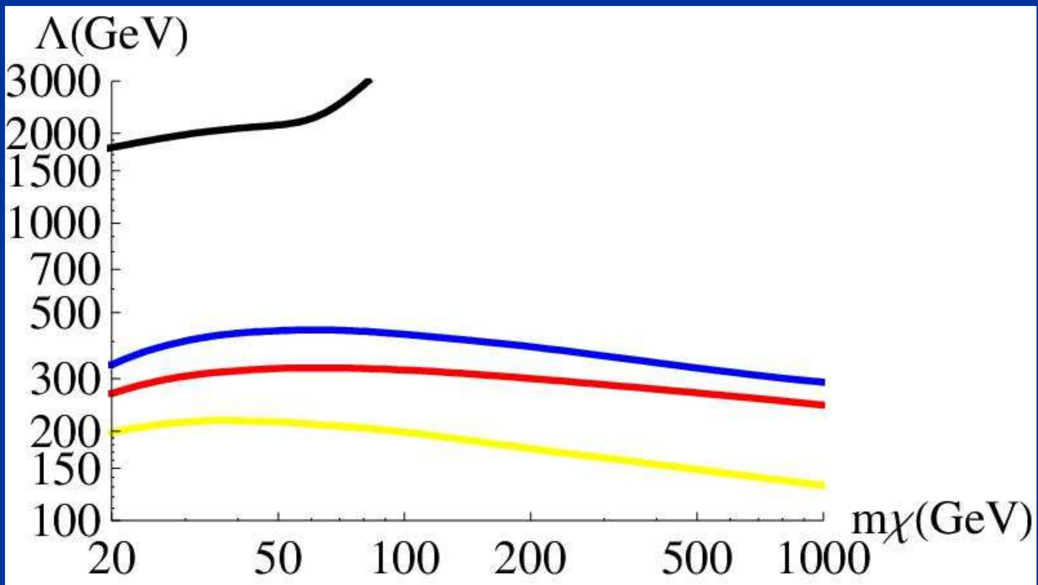
Constraints on C_{WW}

- Interesting interplay between direct detection and LHC searches

XENON1T

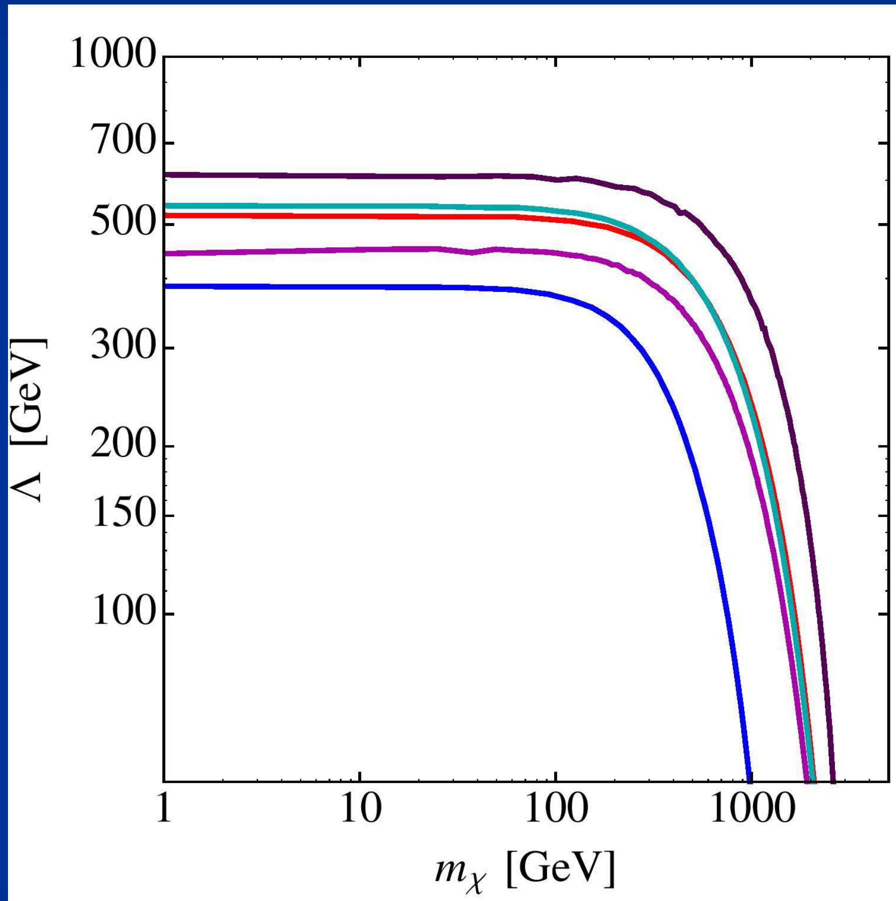
sup erCDMS

LUX



relic density

LHC constraints



preliminary results

Conclusions

- LVF is an excellent place to search for NP
  - $\mu \to e$ conversion sensitive to Higgs mediated flavour violation
- NP cannot explain the current differences in the determination of V_{ub} and V_{cb}
- Interesting loop effects in DM direct detections: new constraints on operators
- EFT provide a consistent framework to search for NO in a model independent way