KING'S College LONDON

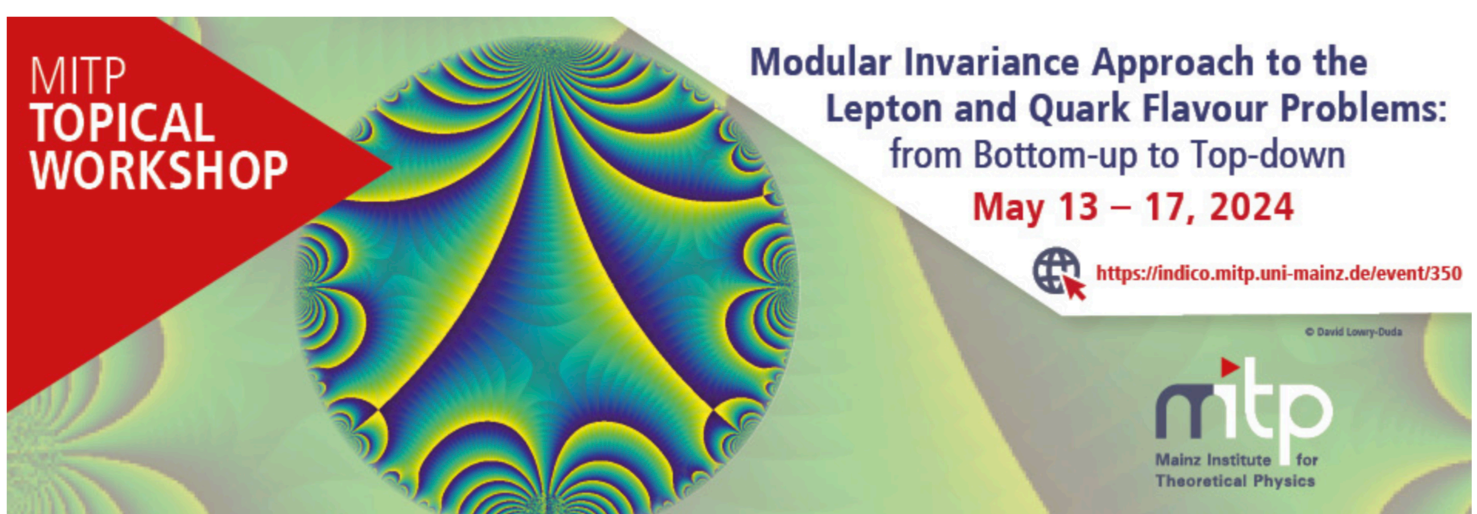
LEVERHULME TRUST

Moduli Stabilization I

Nicole Righi

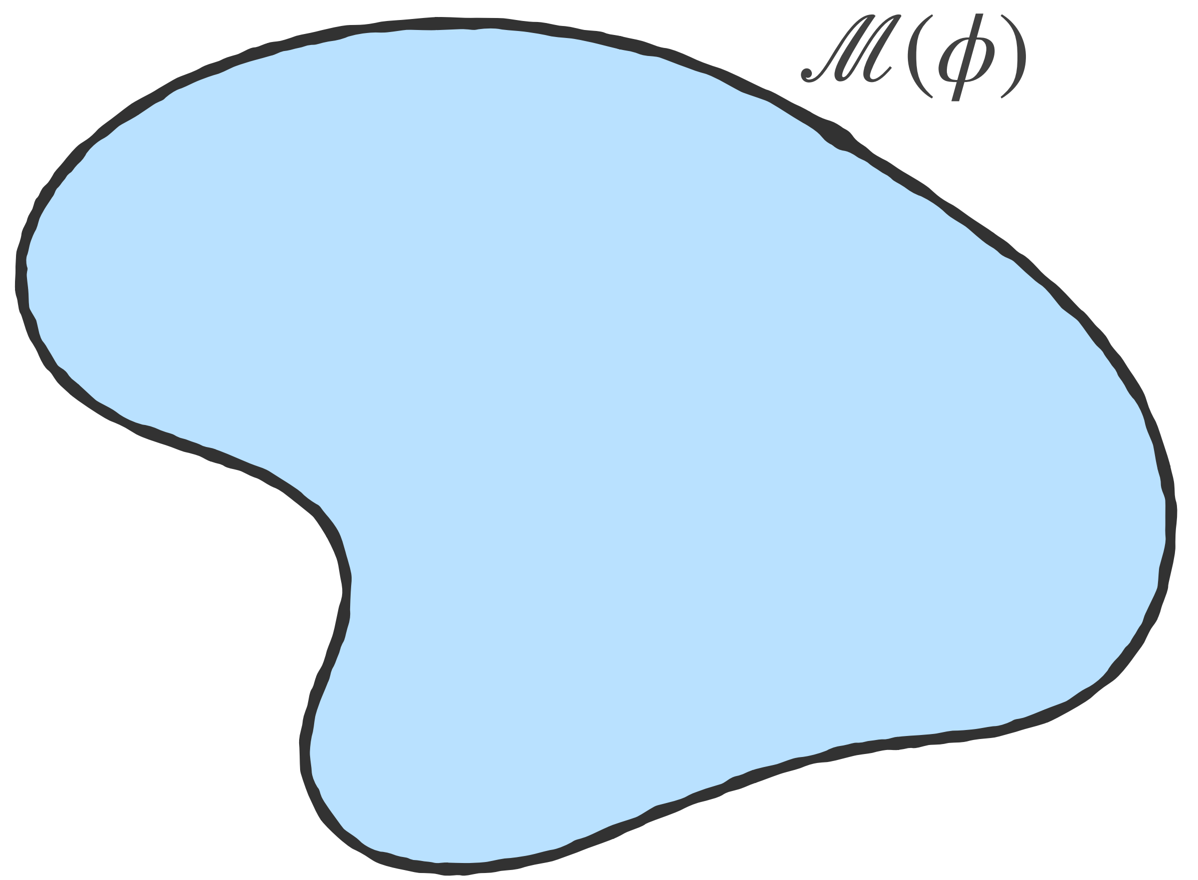
Review talk +

[2212.03876] with J. M. Leedom and A. Westphal and WiP



What is a modulus

Modulus = massless uncharged particle with $V(\phi) = 0$



Why moduli must be heavy

Moduli stabilisation = give a (positive) mass to the moduli

Cosmological moduli problem:

- avoid (unwanted) instabilities during inflation: $m_t^2 \gg H^2$
- avoid fifth forces: $m_t > 10$ TeV
- inconsistencies with cosmological data: decay or dilution

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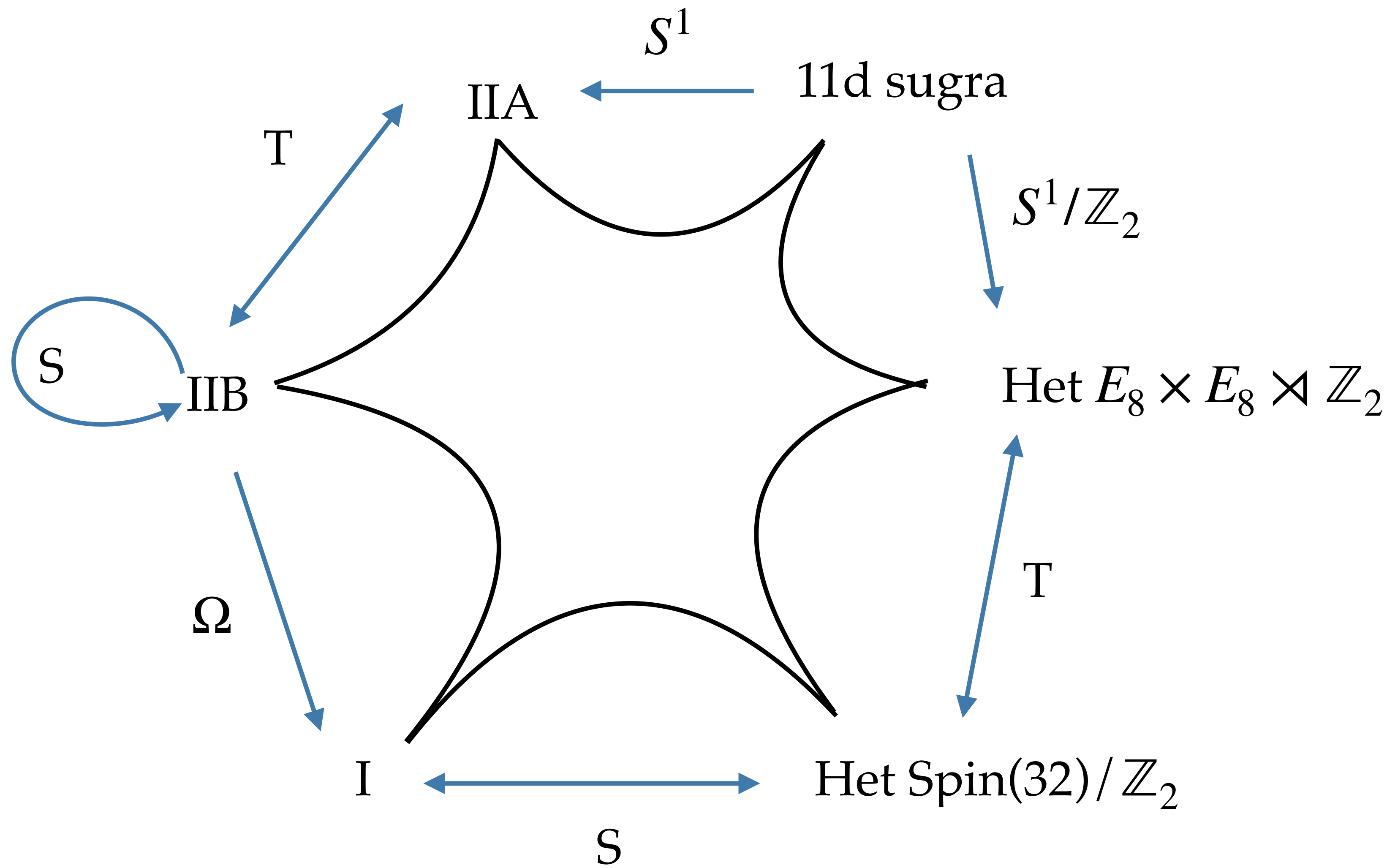
Fundamental moduli problem:

- the moduli determine the geometry of the extra dimensions
- all scales, interactions and couplings come from the geometry of the extra dimensions:

$$M_{string} = \frac{g_s M_{pl}}{\mathcal{V}}$$

- we **must** stabilize moduli to talk about physics scales!

Compactification: why moduli are here



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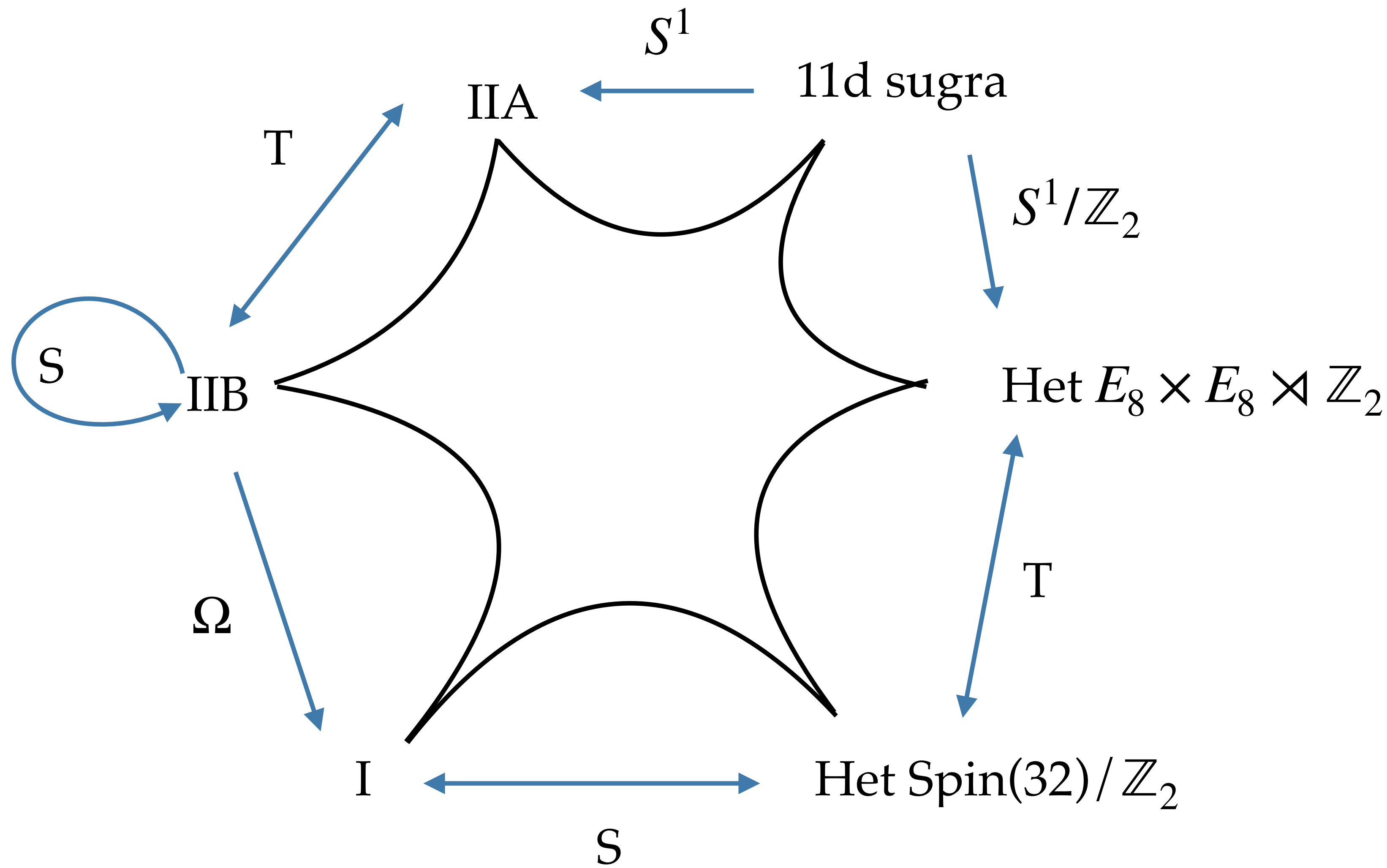

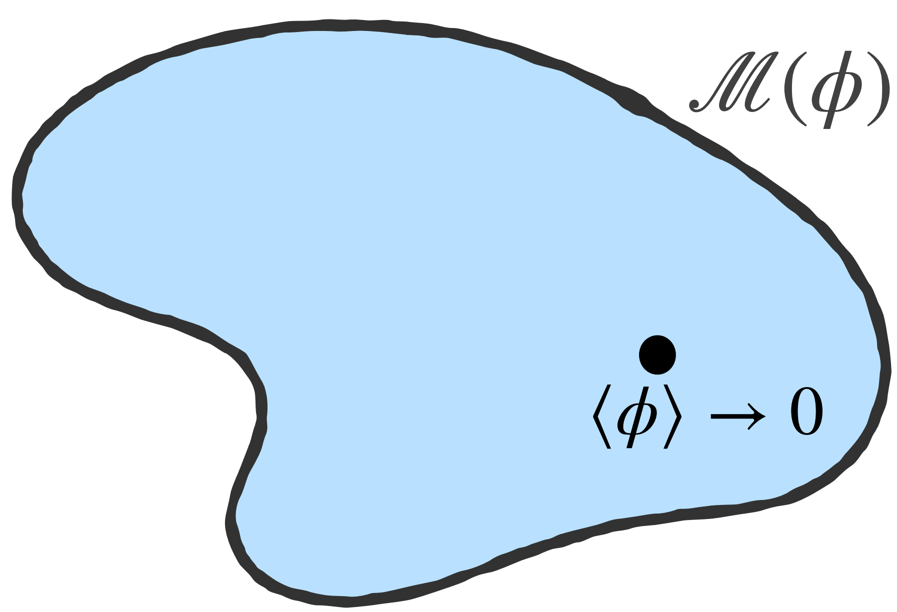

$$g(\phi) \to \infty \quad \longrightarrow \quad \tilde{g} = \frac{1}{g}$$

Compactification: why moduli are here

start with d=10 superstring theory

Compactification: why moduli are here

start with d=10 superstring theory $\longrightarrow$ dilaton e^{φ}

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↓ compactify on a d=6 manifold

get a d=4 effective theory but with too much SUSY

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compactify on a d=6 manifold → metric moduli + axions

- complex structure moduli = shape
- Kahler moduli = size

↓

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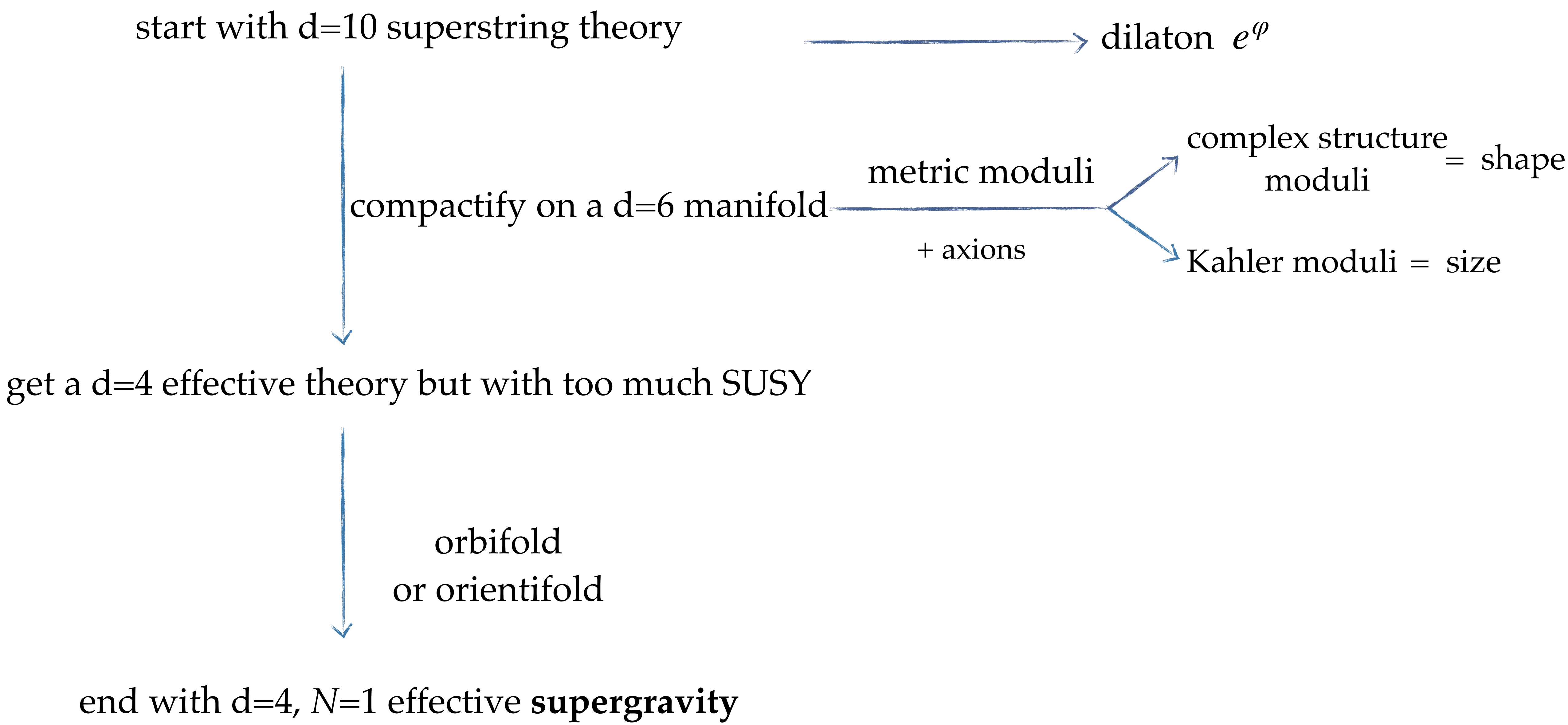

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↓ orbifold or orientifold

end with d=4, N=1 effective **supergravity** + moduli specific of each theory: brane deformation, open string, vector bundle, blow-up moduli….

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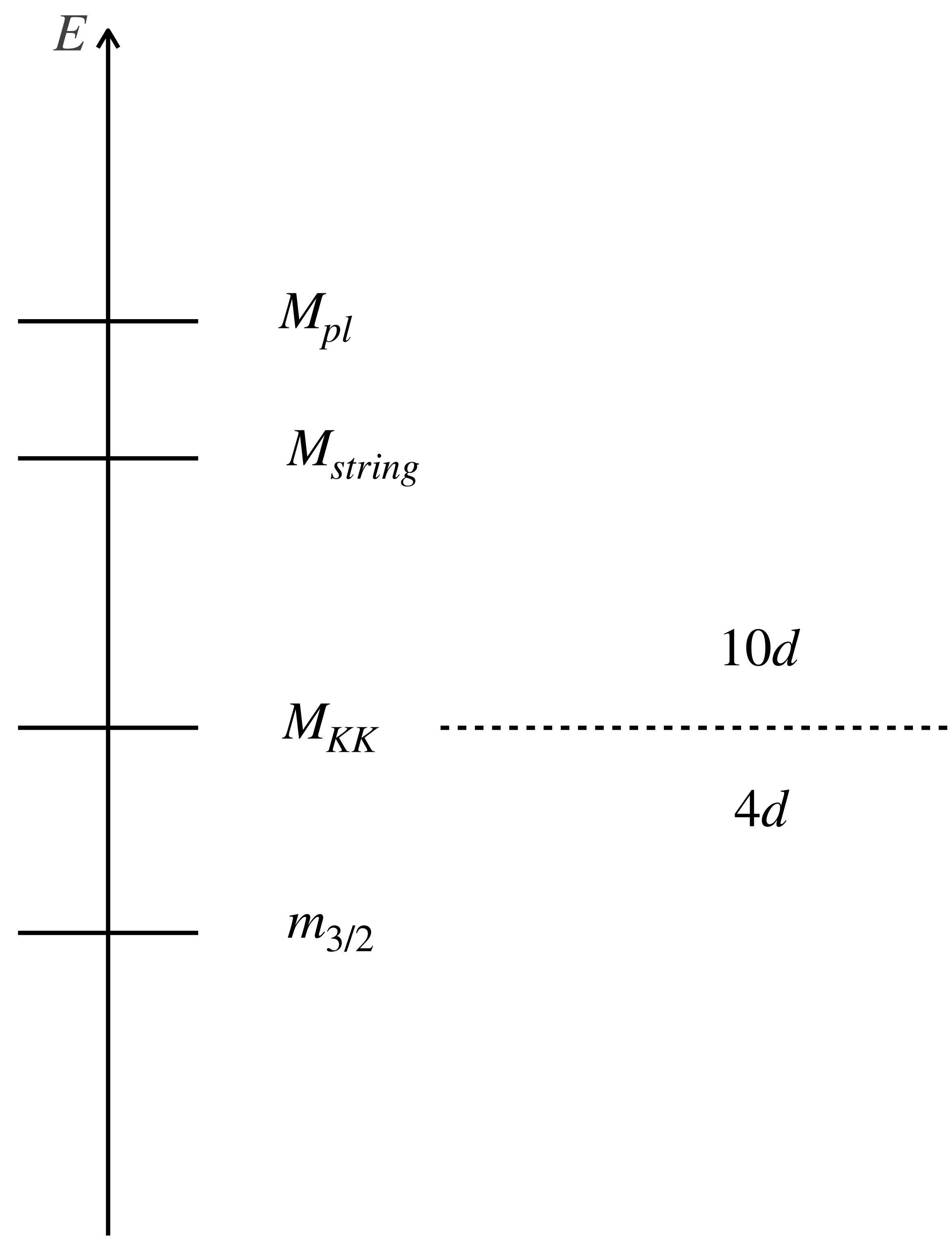
this talk: metric moduli

see Saul and Michael's talk for matter fields from top-down

Some scales



Supergravity

$$V = e^K \left[K^{i\bar{j}} \mathscr{D}_i W \mathscr{D}_{\bar{j}} \overline{W} - 3\,|\,W\,|^2 \right]$$

$$K^{i\bar{j}} = (\partial_i \partial_{\bar{j}} K)^{-1}$$

$$\mathscr{D}_i W = \partial_i W + W \partial_i K \equiv F_i$$ **F-terms**

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- $F_i|_{\langle i \rangle} \neq 0$ susy-breaking

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no-scale property: $V(K_{tree}, W_{tree}) = 0$ see George's talk

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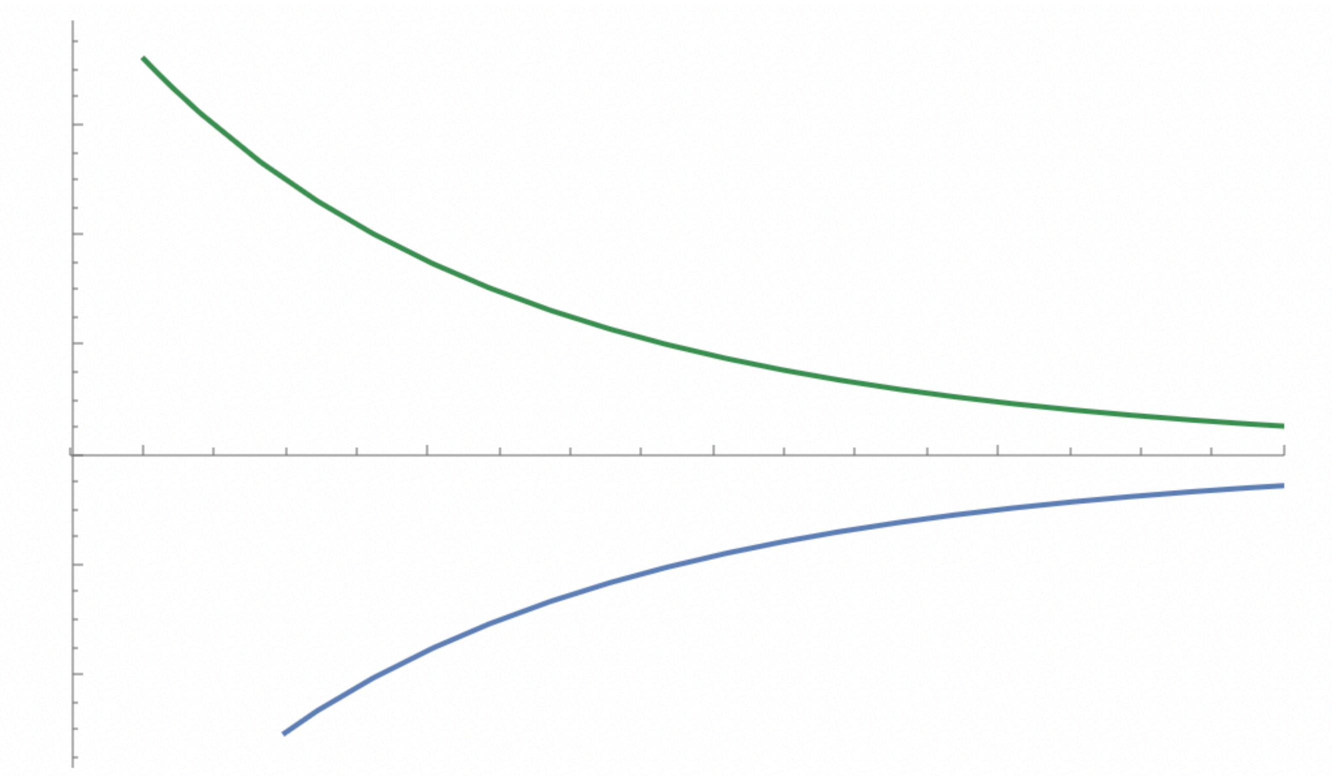
no-scale property: $V(K_{tree}, W_{tree}) = 0$ see George's talk

$\Longrightarrow$ introduce corrections
$$\begin{cases} W = W_{tree} + \delta W_{nonpert} \\ K = K_{tree} + \delta K_{pert} + \delta K_{nonpert} \end{cases}$$

Dine-Seiberg Problem

semiclassical



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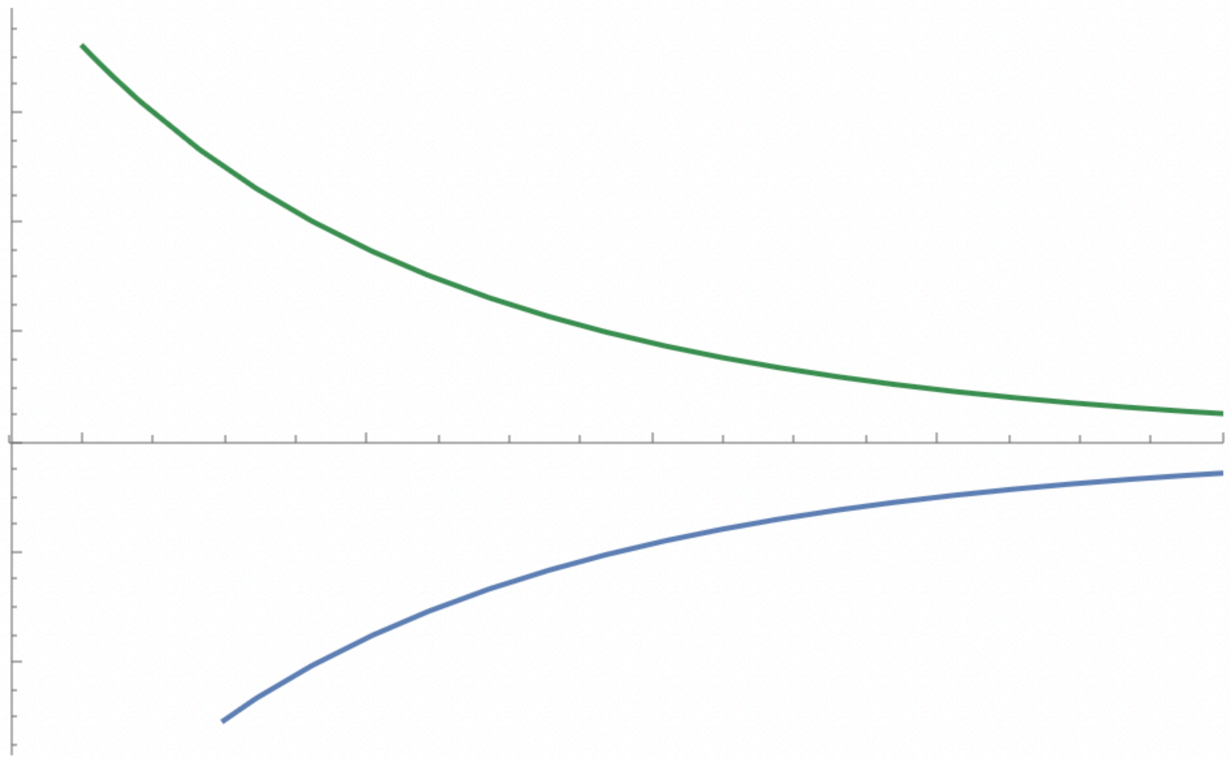
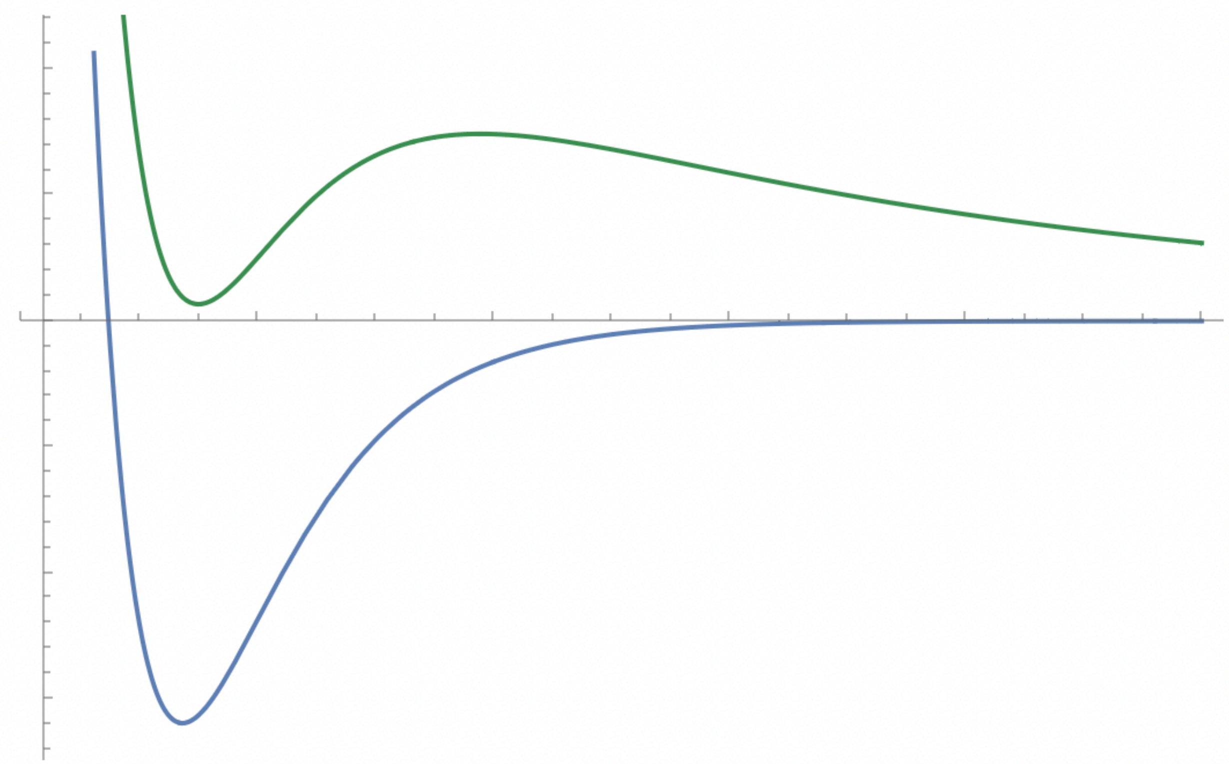


with corrections



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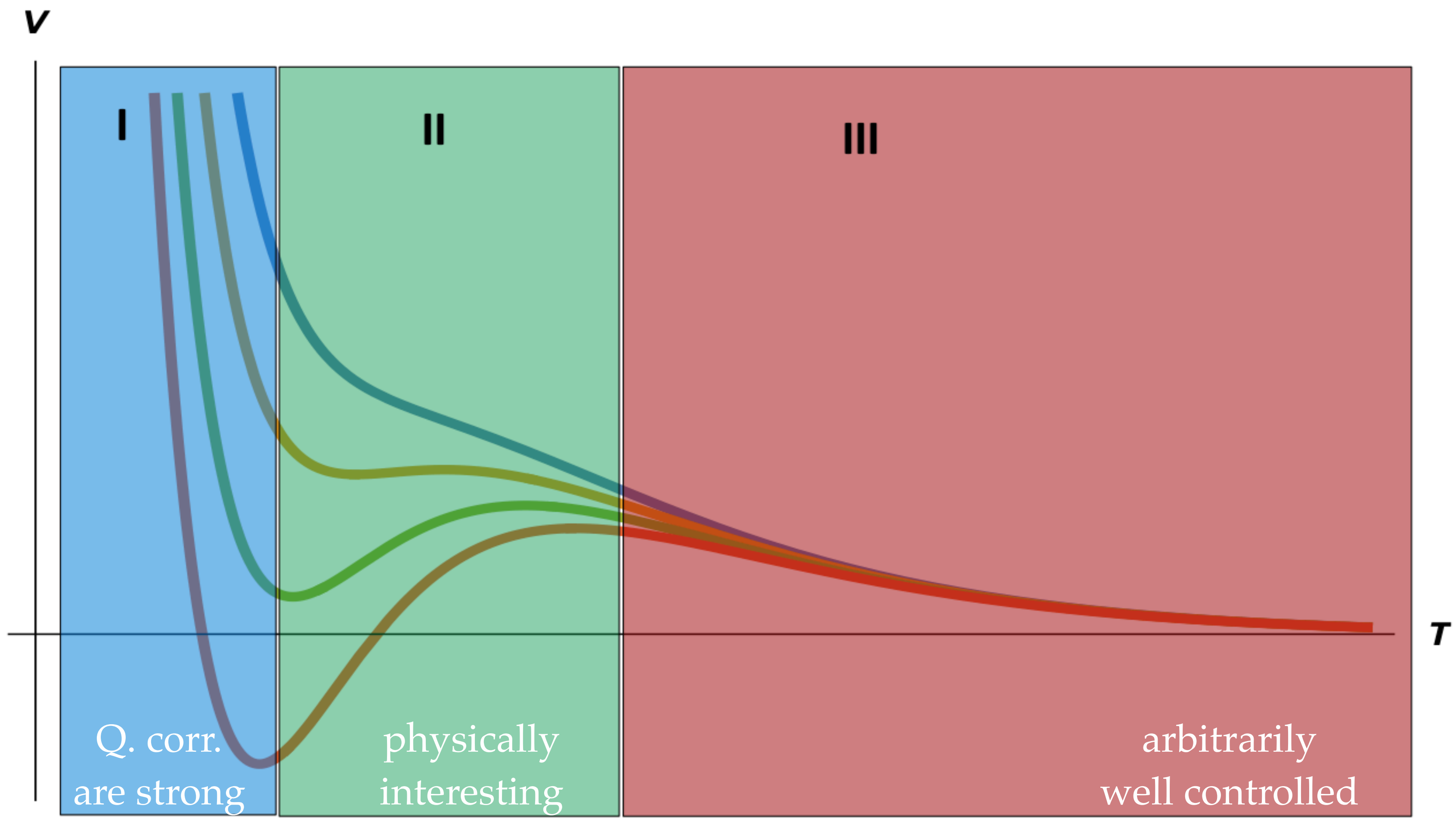
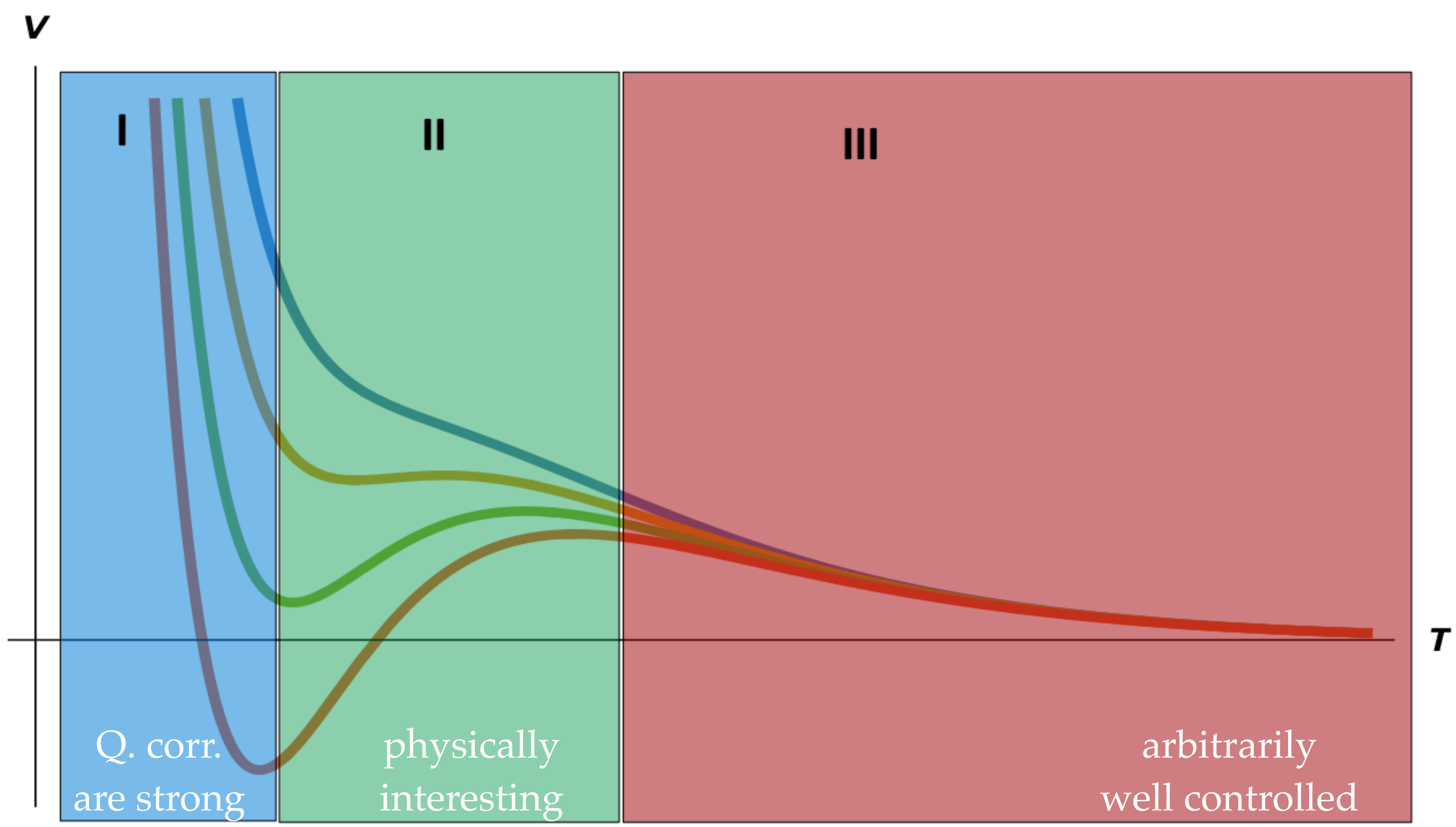


[McAllister, Quevedo, *Moduli Stabilization in String Theory*]

"When corrections can be computed, they are not important, and when they are important, they cannot be computed"

[Denef, *Les Houches Lectures on Constructing String Vacua*]

Dine-Seiberg Problem



[McAllister, Quevedo, *Moduli Stabilization in String Theory*]

e.g. in IIB on CY orientifolds:

$$|W_0| \sim \delta W \gg \delta K$$ [Kachru, Kallosh, Linde, Trivedi '03] KKLT

$$|W_0| \gg \delta K \sim \delta W$$ [Balasubramanian, Berglund, Conlon, Quevedo '05] LVS

$$|W_0| \gg \delta K \gg \delta W$$ [Berg, Haack, Kors '05] most plagued by the problem

→ [Antoniadis, Chen, Leontaris '18]
see George's talk

Origin of the corrections: a (biased) summary

- $K = K_{tree} + \delta K_{pert} + \delta K_{np}$

- $W = W_{tree} + \delta W_{np}$

Origin of the corrections: a (biased) summary

- $K = K_{tree} + \delta K_{pert} + \delta K_{np}$

K_{tree} from the geometry *("reasonably" known)*

$$K_{tree} \sim -2\ln(Vol_{X_6}) - \ln(S+\bar{S}) - \ln\int \Omega \wedge \bar{\Omega}$$

δK_{pert} from 10d higher derivative corrections or string loops contributions *("poorly" known)*

$$\delta K_{pert} \sim -2\ln\left(\frac{\xi(X_6)}{g_s^{3/2}} + \sqrt{g_s}\,\ln(Vol_{X_6})\right) + \dots$$

cf George's talk

δK_{np} from instanton effects *("even more poorly" known)*

$$\delta K_{np} \sim -3\ln\sum_{\mathbf{q}} n_{\mathbf{q}}\left(\mathrm{Li}_3\left(e^{-2\pi\mathbf{q}\cdot\mathbf{t}}\right) + 2\pi\mathbf{q}\cdot\mathbf{t}\,\mathrm{Li}_2\left(e^{-2\pi\mathbf{q}\cdot\mathbf{t}}\right)\right)$$

cf Hajime's talk

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- $W = W_{tree} + \delta W_{np}$

W_{tree} from fluxes *("reasonably" known)*

$$W_{tree} \sim \int_{X_6} G_3 \wedge \Omega$$

δW_{np} from gaugino condensation or branes wrapping cycles *("reasonably" known)*

$$W_{np} \sim e^{-S/b_a} \quad \text{and / or} \quad W_{np} \sim e^{-aT}$$

Positive Λ: a challenge

"there is not a single rigorous 4D de Sitter vacuum in string theory"

[Danielsson, van Riet, *What if string theory has no de Sitter vacua?*]

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Most promising strategy: combine different effects (classical + quantum corrections)

Most challenging side: maintain control over every sector

To compute the effective action one has to make **approximations:** [Baumann, *Inflation in string theory*]

* α' expansion
* String loop expansion
* Probe approximation
* Large charge approximation
* Smeared approximation
* Linear approximation
* Noncompact approximation
* Large volume approximation
* Adiabatic approximation
* Truncation
* Moduli space approximation

Positive Λ: no-go theorems

→ The Landscape: our universe is just one out of many different vacua

← The Swampland: not all the Landscape is consistent in a full theory of QG

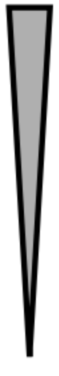
outstanding work on heterotic vacua and SM → No-go thms for de Sitter vacua

⇒ Lessons for the Swampland program

Positive Λ: no-go theorems

$$R_4 + e^{2A}\left(-T^{\mu}_{\mu} + \frac{1}{2}T^{L}_{L}\right) = 2e^{-2A}\nabla^2 e^{2A} \quad \text{iff} \qquad R_4 \leq 0$$

[Maldacena, Nunez '00]



Classical SUGRA?

no dS

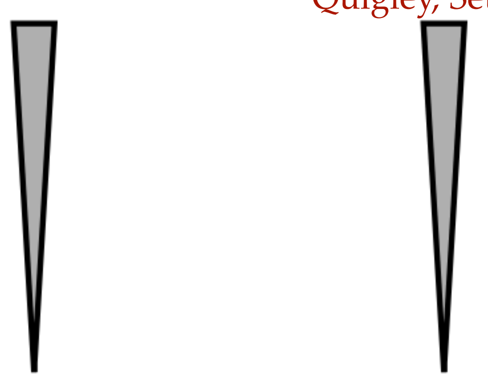
AdS ok

Positive Λ: no-go theorems

$$S = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}\,e^{-2\phi}\left[R + 4(\partial\phi)^2 - \frac{1}{2}|H|^2 - \frac{\alpha'}{4}\left(\mathrm{Tr}|F|^2 - \mathrm{Tr}|R_+|^2\right)\right]$$

[Maldacena, Nunez '00]

[Green, Martinec, Quigley, Sethi '11]



Classical SUGRA? — Leading α'?

no dS — no dS

AdS ok — AdS ok

Positive Λ: no-go theorems

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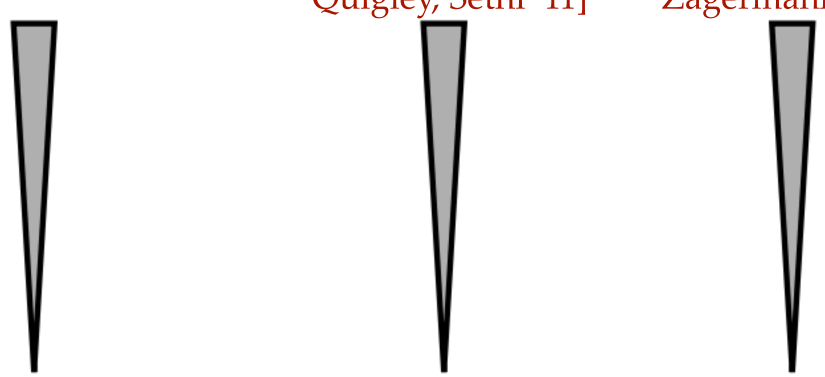
[Maldacena, Nunez '00] [Green, Martinec, Quigley, Sethi '11] [Gautason, Junghans, Zagermann '12]

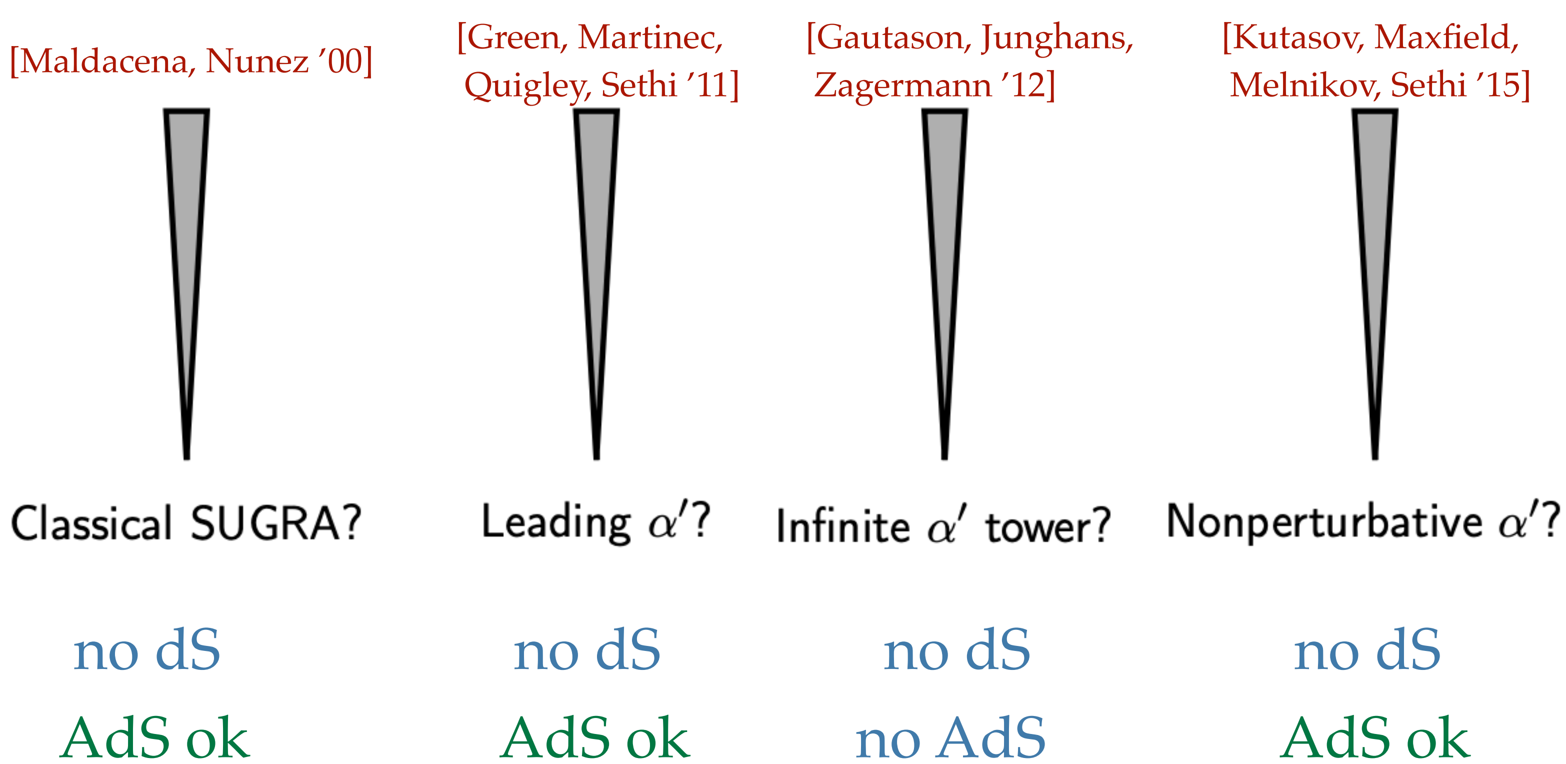


Classical SUGRA? Leading α'? Infinite α' tower?

Classical SUGRA?	Leading α'?	Infinite α' tower?
no dS	no dS	no dS
AdS ok	AdS ok	no AdS

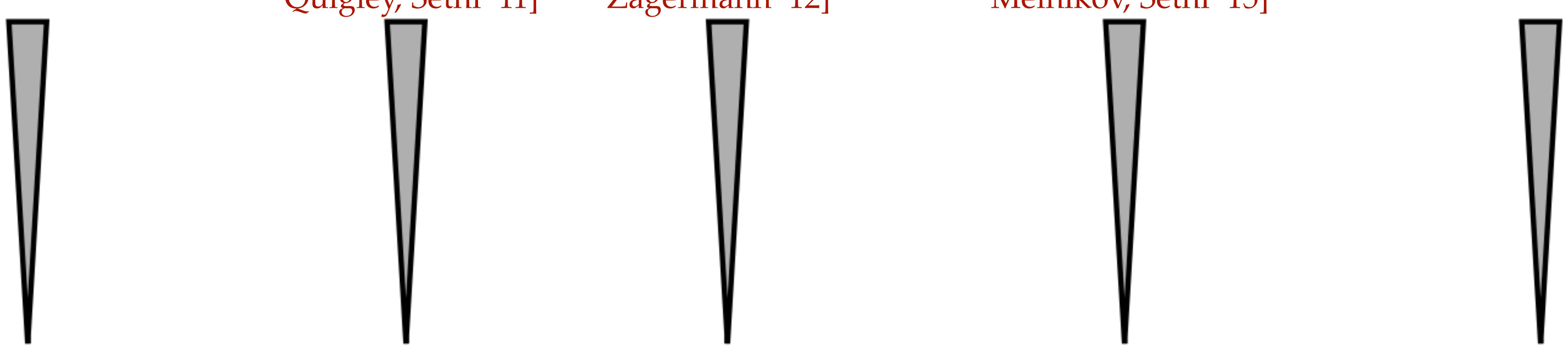
Positive Λ: no-go theorems

Includes tree-level worldsheet & higher derivative corrections



Positive Λ: no-go theorems

$$W(S) \sim e^{-S} \;\rightarrow\; \delta\mathscr{L} \sim e^{-1/g_s^2}$$



[Maldacena, Nunez '00]	[Green, Martinec, Quigley, Sethi '11]	[Gautason, Junghans, Zagermann '12]	[Kutasov, Maxfield, Melnikov, Sethi '15]	[Quigley '15]
Classical SUGRA?	Leading α'?	Infinite α' tower?	Nonperturbative α'?	Nonperturbative g_s, Gaugino Condensation?
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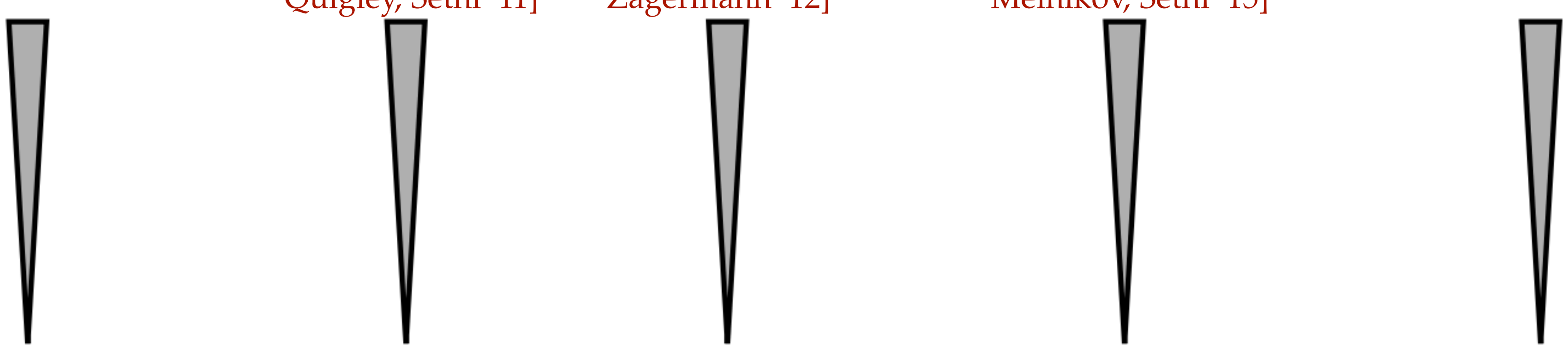

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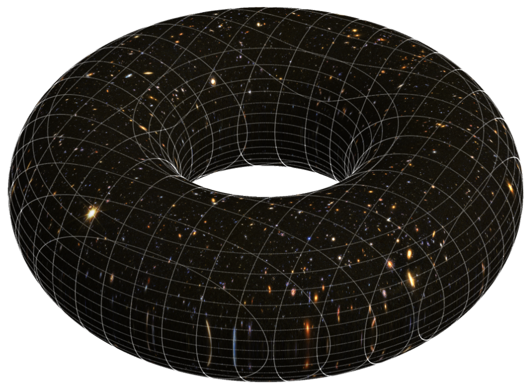
Can we expand these results?

Modular symmetries and heterotic compactification



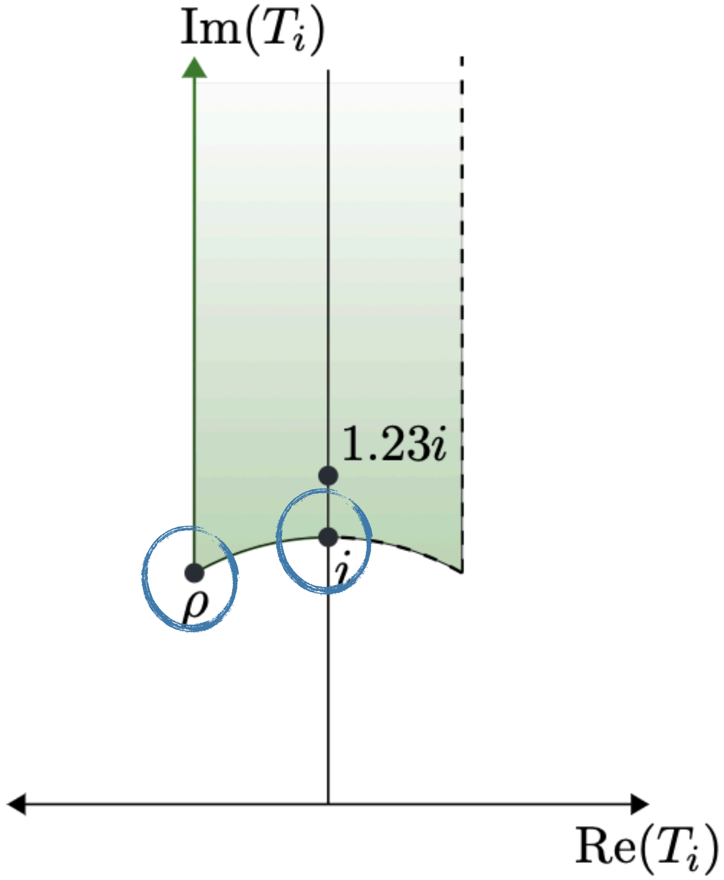
$+ \quad T \leftrightarrow U$

dilaton $S = \dfrac{1}{g_s^2} + i\theta$

Kahler modulus $T = a + it$

Complex structure modulus $U = \dfrac{1}{G_{11}}\left(G_{12} + i\sqrt{\det(G)}\right)$

- T has an $PSL(2,\mathbb{Z})$ symmetry: $T \rightarrow \dfrac{aT+b}{cT+d}$



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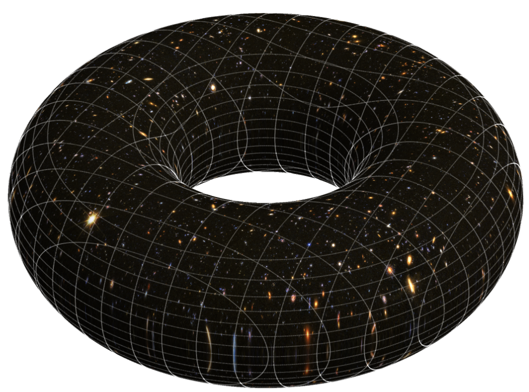

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$$K = -\ln\left(-i(T - \bar{T})\right) \quad \text{with} \quad K \to K + \ln(cT + d) + \ln(c\bar{T} + d)$$

defining $G \equiv K + \ln|W|^2 \quad \Rightarrow \quad V = e^G\left(G_i G^{i\bar{j}} G_{\bar{j}} - 3\right)$

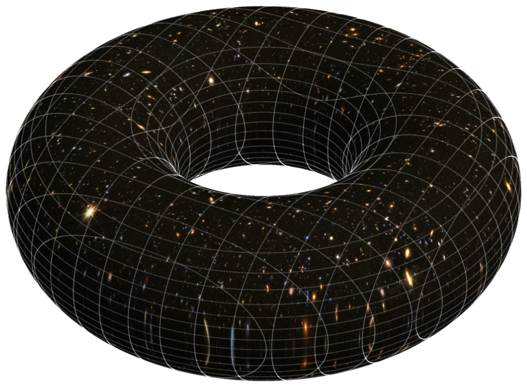
must be a modular function

$\Downarrow$

G must have weight (0,0)

K has weight (1,1) $\quad \Rightarrow \quad$ W must be (-1,0)

Modular symmetries and heterotic compactification



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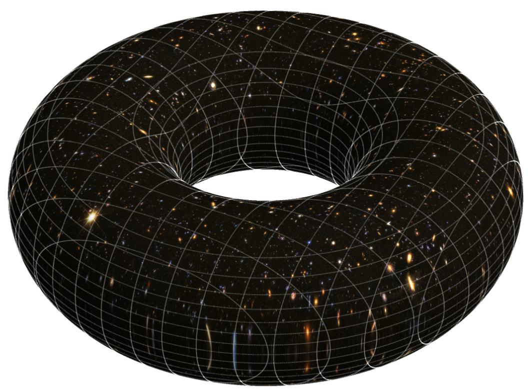
W inherits its automorphic properties from moduli-dependent **threshold corrections**:

Modular symmetries and heterotic compactification



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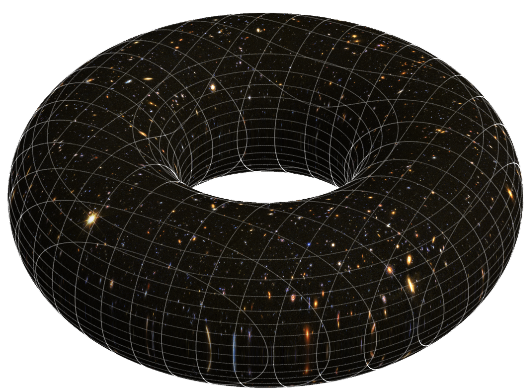

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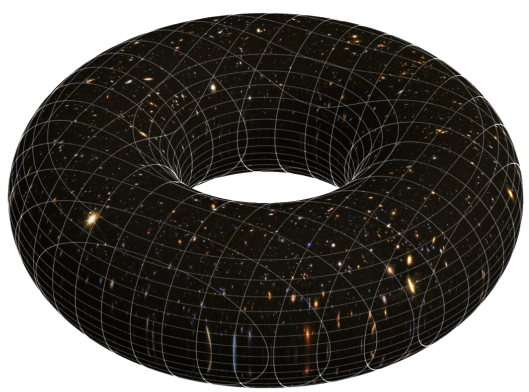
$$f_a = k_a S$$

tree
level

Modular symmetries and heterotic compactification



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$$f_a = k_a S \quad + b_a \ln \eta^2(T) + \text{const.}$$

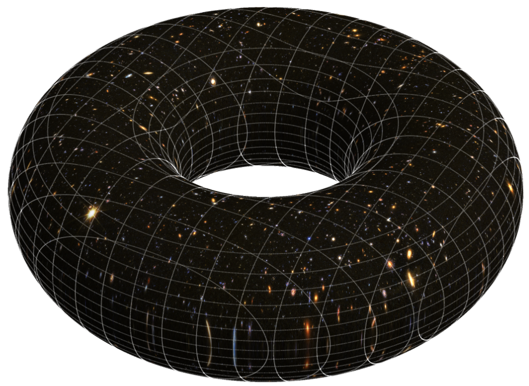
tree level

threshold corrections

[Dixon, Kaplunovsky, Louis '91]

[Kaplunovsky, Louis '95]

Modular symmetries and heterotic compactification



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tree level

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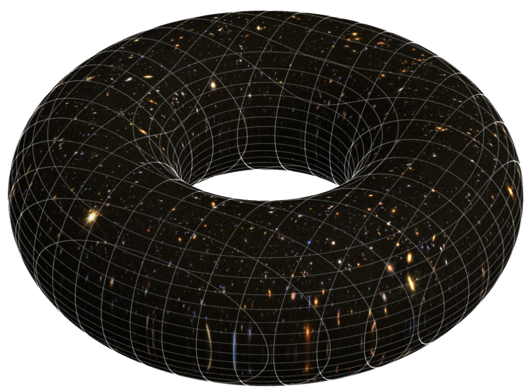
$$\Rightarrow \quad W \sim \frac{e^{-k_a S/b_a}}{\eta(T)^2}$$

Modular symmetries and heterotic compactification



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+ nonperturbative effects in the geometric moduli:

$$W = \frac{\Omega(S)H(T)}{\eta^2(T)}$$

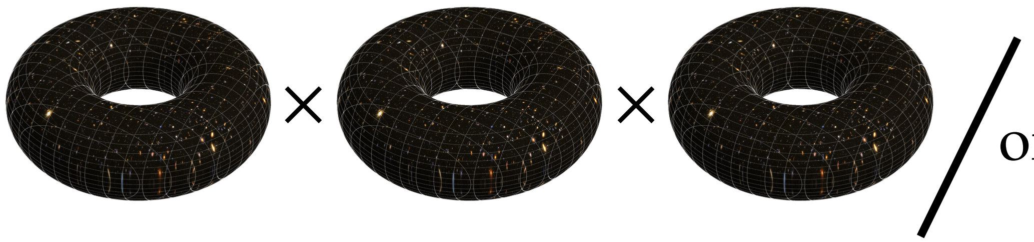
$$\Omega(S) \sim e^{-S}$$

$$H(T) = \left(\frac{G_4(T)}{\eta^8(T)}\right)^n \left(\frac{G_6(T)}{\eta^{12}(T)}\right)^m \mathscr{P}(j(T))$$

$H(T)$ is the **most general modular function** on $SL(2,\mathbb{Z})$ [Rademacher, Zuckerman '38]

Two moduli: ST model

[Font, Ibanez, Lüst, Quevedo '90]
[Cvetic, Font, Ibanez, Lüst, Quevedo '91]



orbifold

- Kahler potential $K = -k(S,\bar{S}) - 3\ln\left(-i(T-\bar{T})\right)$

- superpotential $W = \dfrac{\Omega(S)H(T)}{\eta^6(T)}$

$$\Omega(S) \sim e^{-S}$$

$$H(T) = \left(\frac{E_4(T)}{\eta^8(T)}\right)^n \left(\frac{E_6(T)}{\eta^{12}(T)}\right)^m \mathscr{P}(j(T))$$

Two moduli: ST model

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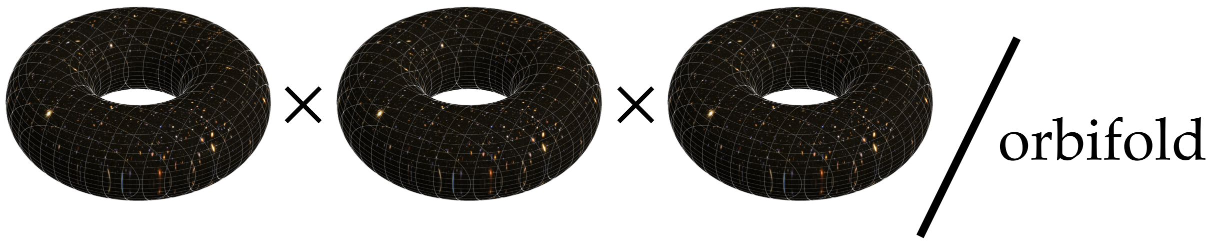

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- Scalar potential

$$V(T,S) = e^{k(S,\bar{S})}\,Z(T,\bar{T})\,|\Omega(S)|^2\,\left[\left(A(S,\bar{S})-3\right)|H(T)|^2 + \widehat{V}(T,\bar{T})\right]$$

Two moduli: *ST* model

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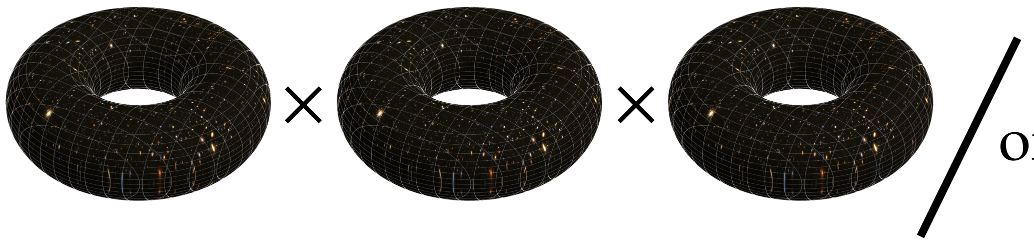

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- Scalar potential

$$V(T,S) = e^{k(S,\bar{S})} \underbrace{Z(T,\bar{T})}_{>0} |\Omega(S)|^2 \left[\left(\underbrace{A(S,\bar{S})}_{\sim |F_S|^2 \geq 0} - 3\right)|H(T)|^2 + \underbrace{\widehat{V}(T,\bar{T})}_{>0}\right]$$

F-terms $F_S = \partial_S W + W\partial_S K = \dfrac{H(T)}{\eta^6(T)}\left(\dfrac{d\Omega}{dS} + \dfrac{dk(S,\bar{S})}{dS}\Omega\right)$

see [King, Wang '23] and Xin's talk for a generalisation

New de Sitter no-go theorem

$$V(T,S) = e^{k(S,\bar{S})} Z(T,\bar{T}) \, |\Omega(S)|^2 \left[\left(A(S,\bar{S}) - 3 \right) |H(T)|^2 + \widehat{V}(T,\bar{T}) \right]$$

$\sim |F_S|^2 \geq 0$

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Two classes of extrema:

- $F_S = 0$
- $F_S \neq 0$

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  (ii). $\partial_S V(T_0, S_0) = 0 \quad \& \quad \partial_T V(T_0, S_0) = 0$

  (iii). $(\Omega_S + k_S \Omega)|_{S=S_0} = 0$

  (iv). Eigenvalues of the Hessian of $V(T,S)$ at (T_0, S_0) are all ≥ 0

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$\partial_T V$ is a weight 2 modular form $\Rightarrow$ $\partial_T V|_{T=i,\rho} = 0$

$\partial_S \partial_T V$ is a weight 2 modular form $\Rightarrow$ $\partial_S \partial_T V|_{T=i,\rho} = 0$

Hessian is block diagonal

$\Rightarrow$ when are $T = i, T = \rho$ **dS** minima?

Modular landscape of vacua

- Self dual points: $W = \dfrac{\Omega(S)H(T)}{\eta^6(T)}$ where $H(T) = \left(\dfrac{E_4(T)}{\eta^8(T)}\right)^n \left(\dfrac{E_6(T)}{\eta^{12}(T)}\right)^m \mathscr{P}(j(T))$

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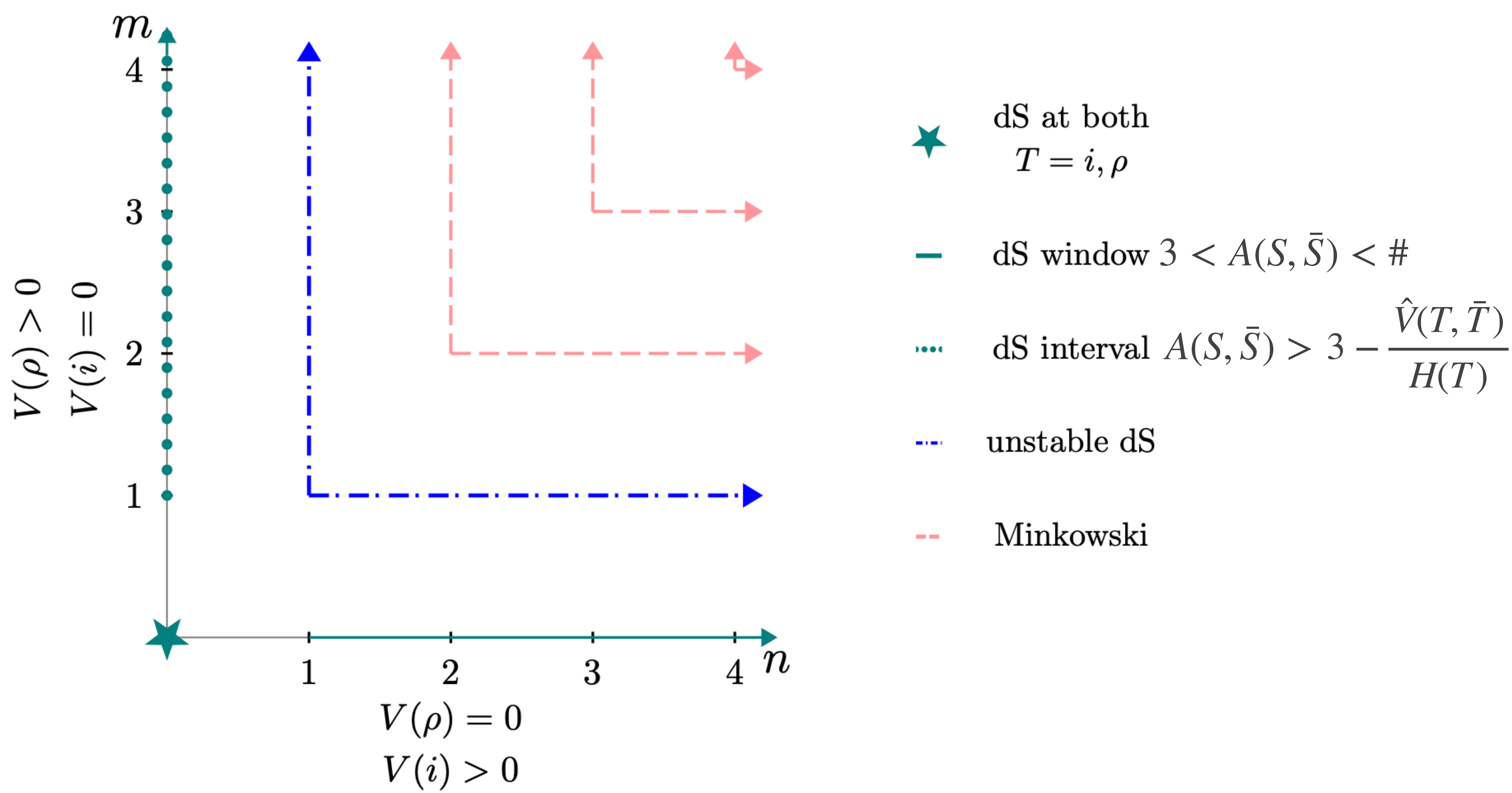
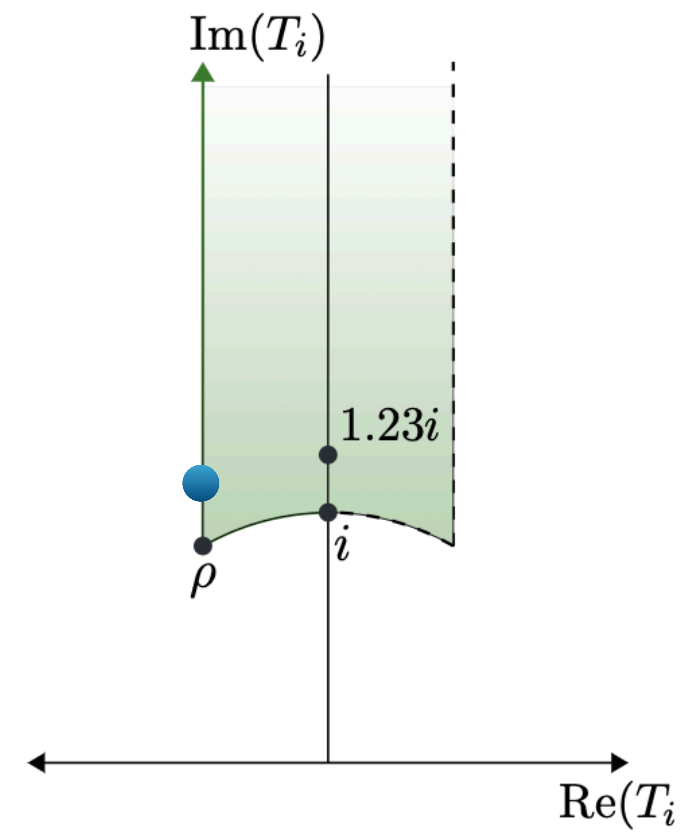
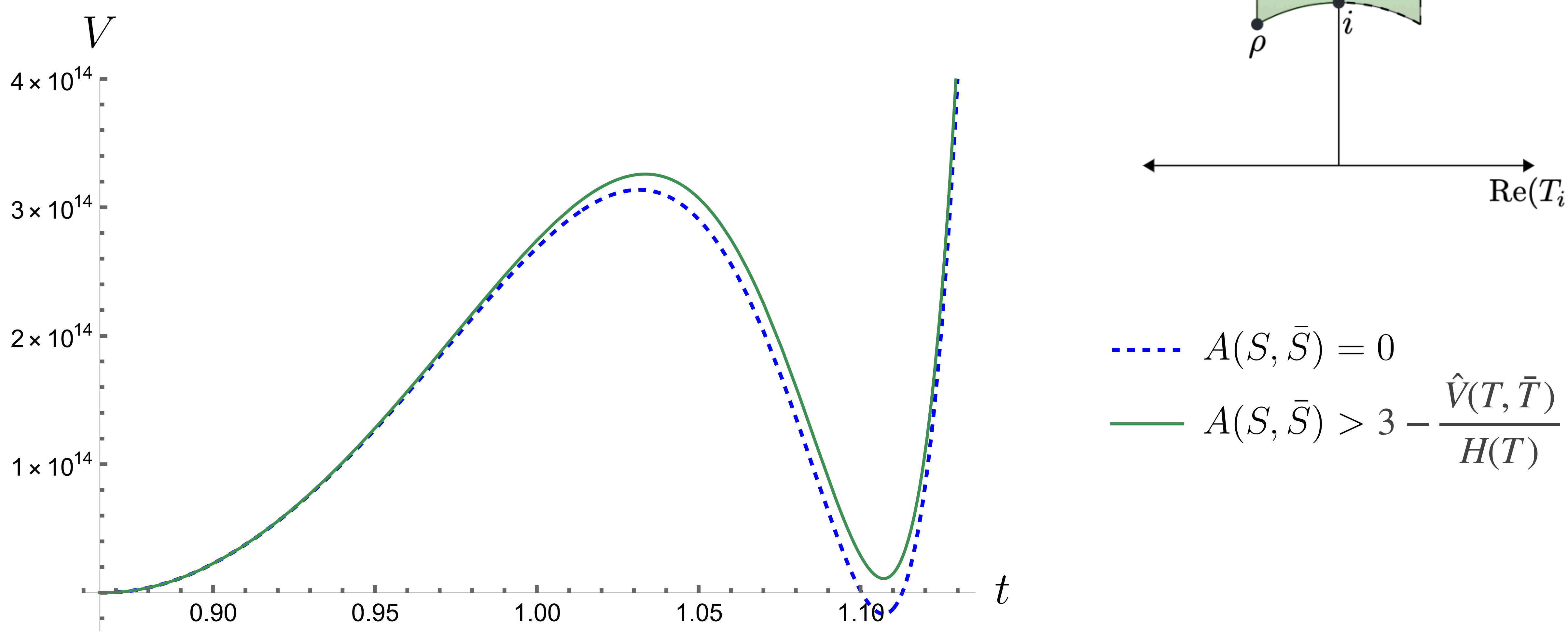


Modular landscape of vacua

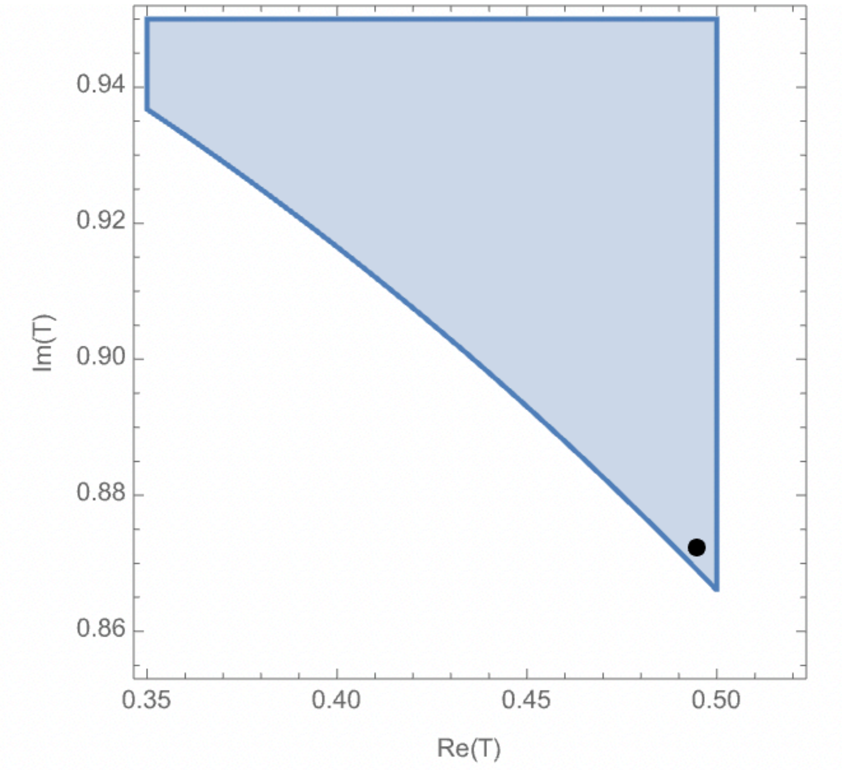
- Into the fundamental domain: an example



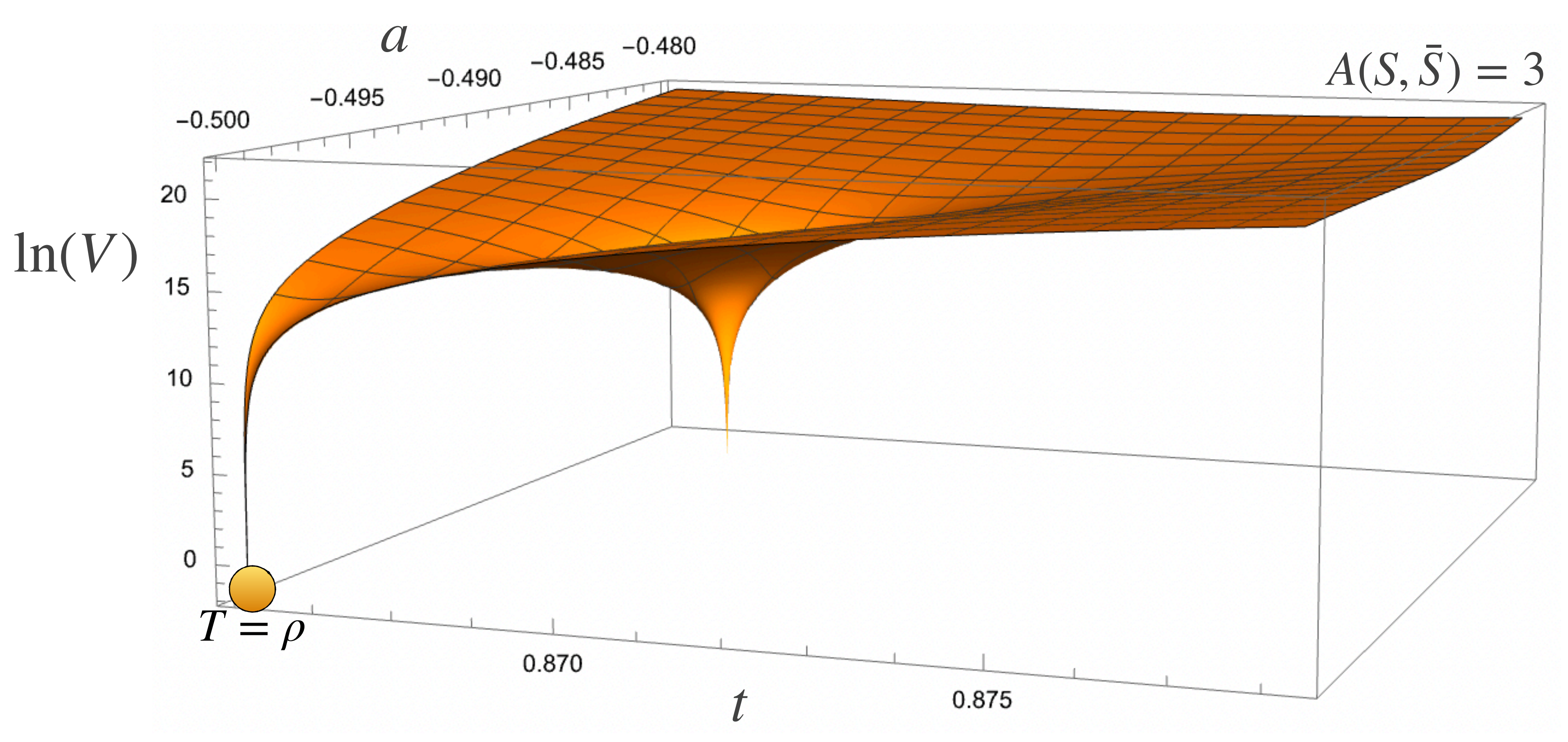


Modular landscape of vacua

- Into the fundamental domain: symmetry-breaking points



[Novichkov, Penedo, Petcov '22]
see Joao's talk



Stringy instantons and de Sitter

How can we realise $A(S,\bar{S}) > 3 - \dfrac{\hat{V}(T,\bar{T})}{H(T)}$?

Stringy instantons and de Sitter

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$\longrightarrow$ no dS minima for $F_S \neq 0$ and $K = -\ln(S+\bar{S})$

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⟶ $K = -\ln(S+\bar{S}) + \delta K_{np}$

Stringy instantons and de Sitter

→ no dS minima for $F_S \neq 0$ and $K = -\ln(S + \bar{S})$

↪ $K = -\ln(S + \bar{S}) + \delta K_{np}$

All closed string theories have nonperturbative contributions of strength $\sim e^{-1/g_s}$

[Shenker '90]

Stringy instantons and de Sitter

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↳ $K = -\ln(S+\bar{S}) + \delta K_{np}$

All closed string theories have nonperturbative contributions of strength $\sim e^{-1/g_s}$

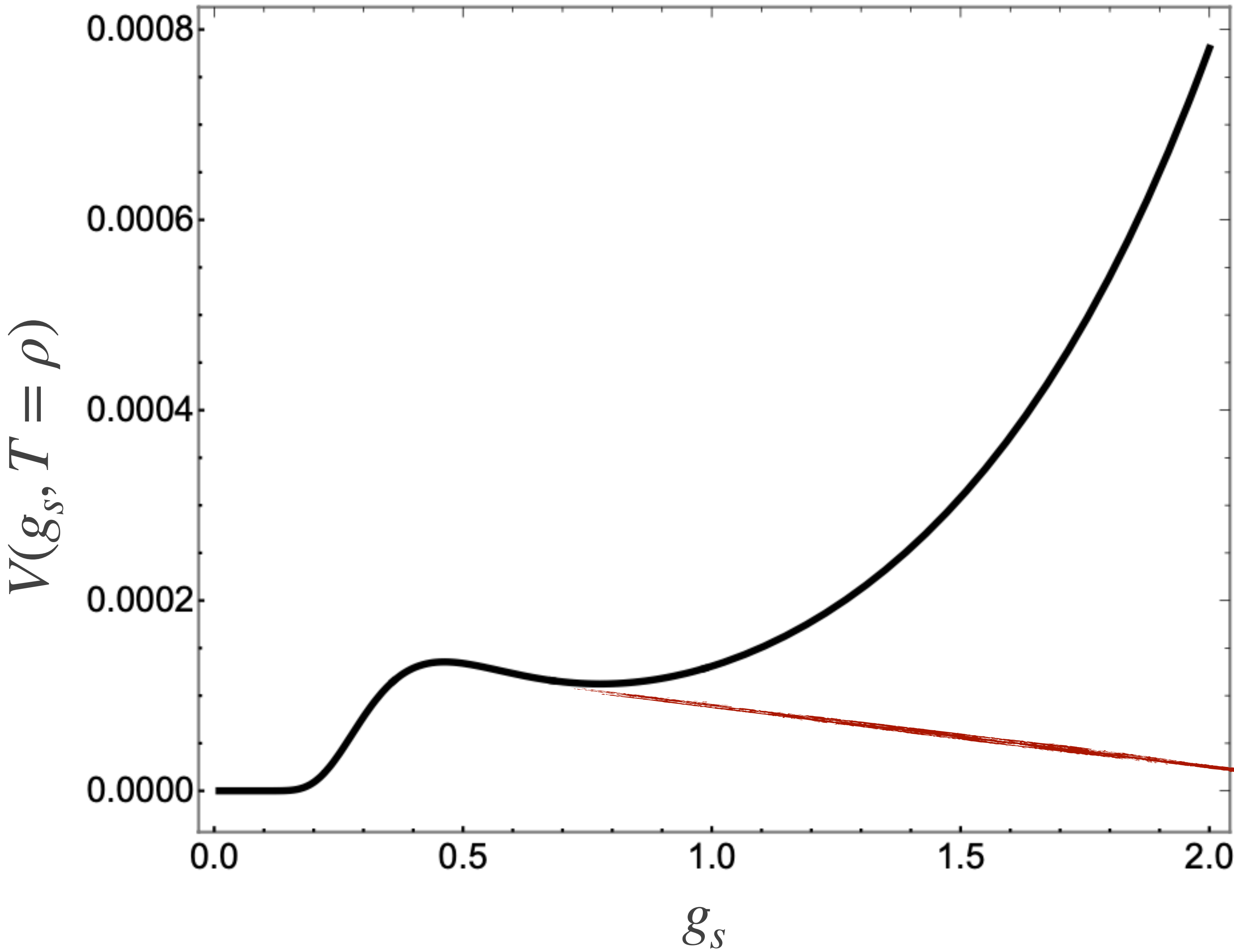
[Shenker '90]

→ Quite odd in heterotic — no D-branes!

Dig in the literature: very few studies

- String dualities $\Rightarrow$ corrections to Kahler potential [Silverstein '94]
- Present in 10d and 9d in heterotic SO(32)
  Needed for SO(32)/type I duality to hold! [Polchinski, '05] [Green, Rudra '16]

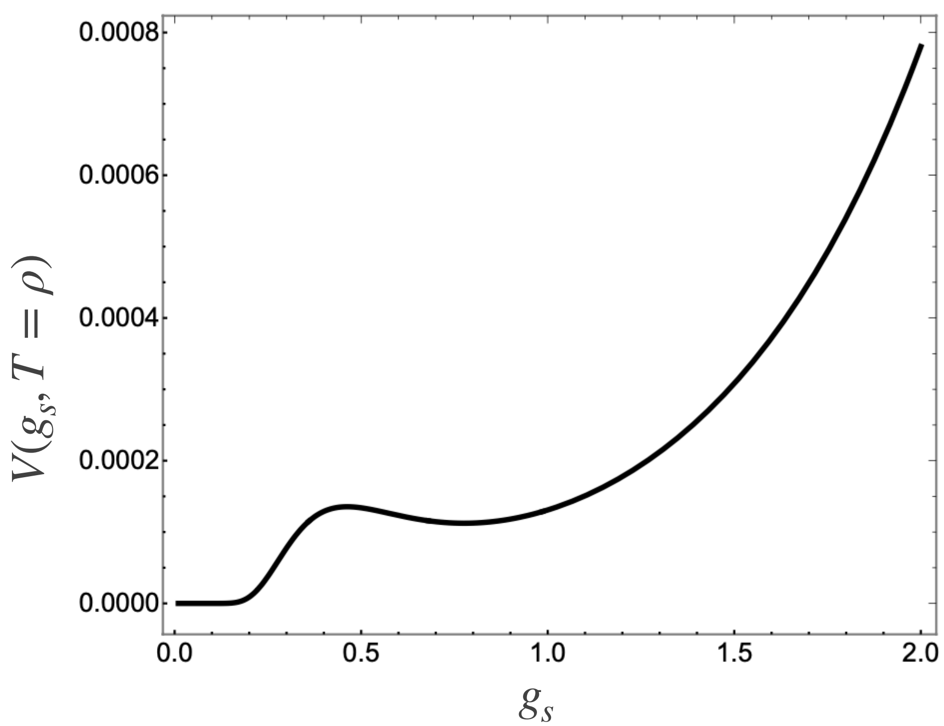
A working example



metastable dS vacuum

$\langle g_s \rangle \simeq 0.7$

A working example



metastable dS vacuum

↓

what generates these corrections?

Wip

TBD

- extend the construction to more fields
- understand the nature of nonperturbative effects
- more on heterotic on orbifolds and the swampland

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-
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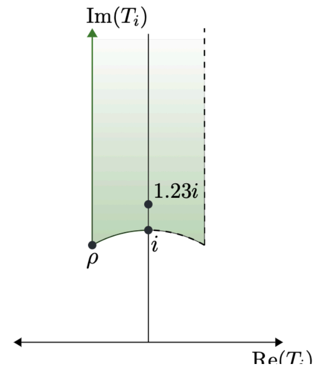
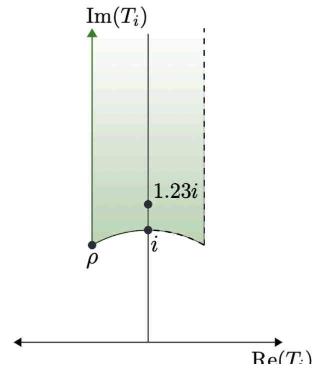
SL(2,$\mathbb{Z}$)

Fundamental domain	 with 2 fixed pts	
Modular form of weight k	$f(\gamma \cdot x) = (cx + d)^k f(x)$ $\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z})$ $ad - bc = 1$	
Ring	E_4, E_6, η	

	SL(2,ℤ)	Sp(4,ℤ)
Fundamental domain	 with 2 fixed pts	Siegel upper half plane with 6 fixed pts σ_i cf. Xiang-Gan's talk
Modular form of weight k	$f(\gamma \cdot x) = (cx+d)^k f(x)$ $\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z})$ $ad - bc = 1$	$f(\Gamma \cdot X) = \det(CX+D)^k f(X)$ $\Gamma = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \in \mathrm{Sp}(4,\mathbb{Z})$ $AD - BC = 1$ cf. Saul's talk
Ring	E_4, E_6, η	$\mathscr{E}_4, \mathscr{E}_6, \chi_{10}, \chi_{12}, \chi_{35}$

	SL(2,$\mathbb{Z}$)	Sp(4,$\mathbb{Z}$)
	$K = -\log[-\frac{i}{2}(T - \bar{T})]$	$K_{(2)} = -\ln[-\frac{1}{4}\det(M - M^{\dagger})]$ $= -\ln[-\frac{1}{4}(T - \bar{T})(U - \bar{U}) + \frac{1}{4}(V - \bar{V})^2]$ [Lopes Cardoso, Lüst, Mohaupt '94]

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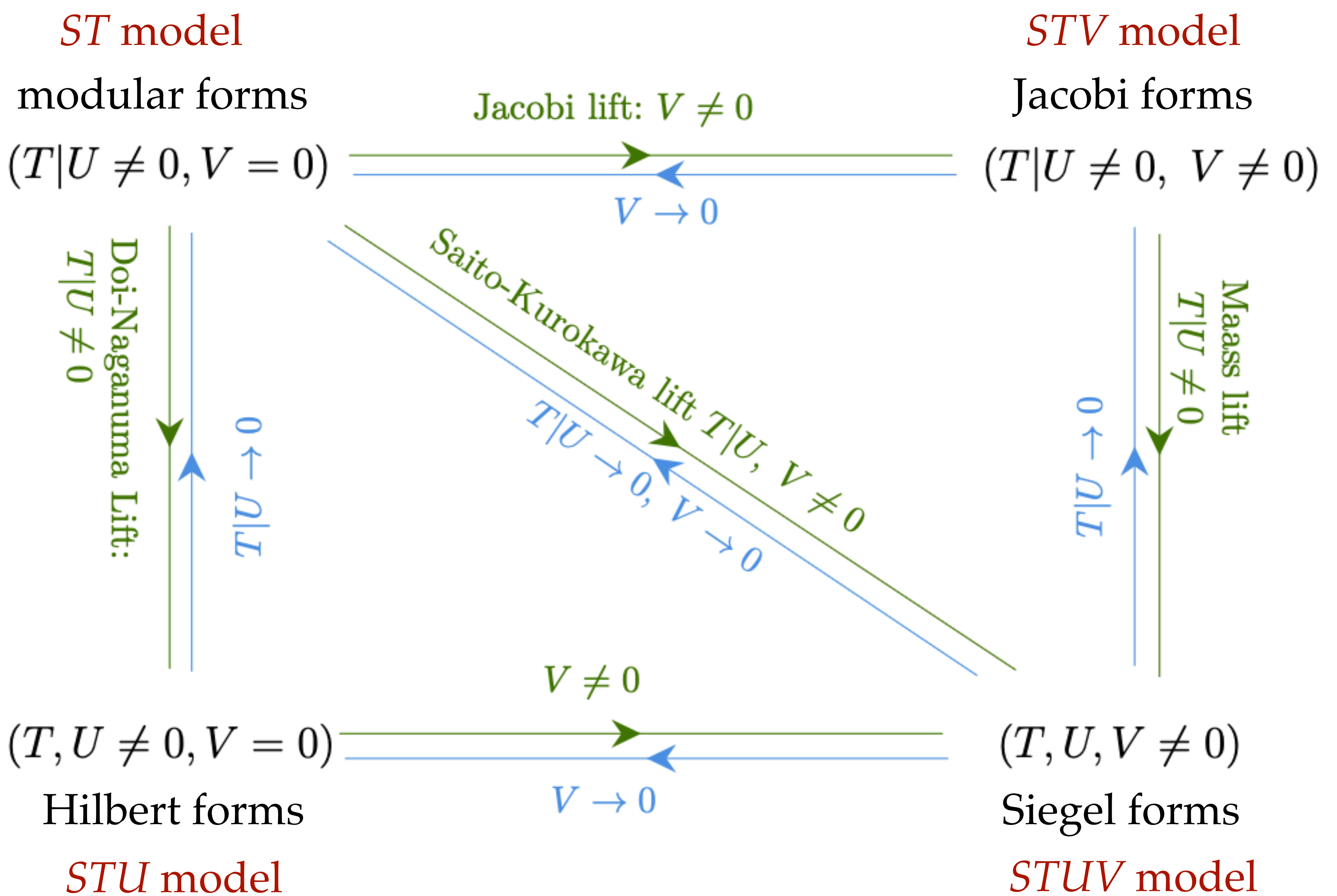
	$W \sim \frac{e^{-S} H(T)}{\eta^6(T)}$	$W_{(2)} \sim \frac{e^{-S} H_{(2)}}{\chi_{12}}$ [Mayr, Stieberger '95] [Stieberger '98]

	$SL(2,\mathbb{Z})$	$Sp(4,\mathbb{Z})$
	$K = -\log[-\frac{i}{2}(T-\bar{T})]$	$K_{(2)} = -\ln[-\frac{1}{4}\det(M-M^\dagger)]$ $= -\ln[-\frac{1}{4}(T-\bar{T})(U-\bar{U}) + \frac{1}{4}(V-\bar{V})^2]$ [Lopes Cardoso, Lüst, Mohaupt '94]
	$W \sim \frac{e^{-S}H(T)}{\eta^6(T)}$	$W_{(2)} \sim \frac{e^{-S}H_{(2)}}{\chi_{12}}$ [Mayr, Stieberger '95] [Nilles, Stieberger '97] [Stieberger '98]
	$H(T) = \left(\frac{E_4(T)}{\eta^8(T)}\right)^n \left(\frac{E_6(T)}{\eta^{12}(T)}\right)^m \mathscr{P}(j(T))$	$H_{(2)} = \left(\frac{\mathscr{E}_4^3}{\chi_{12}}\right)^n \left(\frac{\mathscr{E}_6^2}{\chi_{12}}\right)^m \left(\frac{\mathscr{E}_4^2\mathscr{E}_6\chi_{10}}{\chi_{12}^2}\right)^\ell \mathscr{P}(j_{(2)})$ [Kidambi, Leedom, NR, Westphal *WiP*]

Lifts and truncations: the check

$$H_{(2)} = \left(\frac{\mathscr{E}_4^3}{\chi_{12}}\right)^n \left(\frac{\mathscr{E}_6^2}{\chi_{12}}\right)^m \left(\frac{\mathscr{E}_4^2\mathscr{E}_6\chi_{10}}{\chi_{12}^2}\right)^\ell \mathscr{P}(j_{(2)}) \quad \longrightarrow \quad H = \left(\frac{E_4}{\eta^8}\right)^n \left(\frac{E_6}{\eta^{12}}\right)^m \mathscr{P}(j)$$



Preliminary: the potential

without explicitly computing $V(S, T, U, V)$, we *prove*:

- all 6 fixed points σ_i are extrema:

  since $V(S, T, U, V)$ is a Siegel modular function, $\nabla V(S, T, U, V)|_{\{T,U,V\}=\sigma_i} = 0$

- these extrema are always either Minkowski or AdS minima when $F_S = 0$

[Kidambi, Leedom, NR, Westphal *WiP*]

$\Rightarrow$ can we uplift requiring $F_S \neq 0$?

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[Kidambi, Leedom, NR, Westphal *WiP*]

$\Rightarrow$ can we uplift requiring $F_S \neq 0$?

For $F_S \neq 0$, at the 2 fixed pts: $H(T) = \left(\frac{E_4(T)}{\eta^8(T)}\right)^n \left(\frac{E_6(T)}{\eta^{12}(T)}\right)^m \mathscr{P}(j(T))$ tell us about the nature of the extrema

$\Rightarrow$ does this happen for $H_{(2)}$ as well?

TBD

- extend the construction to more fields
- understand the nature of nonperturbative effects
- more on heterotic on orbifolds and the swampland

- extend the extrema analysis made for $SL(2,\mathbb{Z})$ to $Sp(4,\mathbb{Z})$
  - $\longrightarrow$ new, bigger landscape of heterotic vacua
  - $\longrightarrow$ extension of the no-go theorems: new dS vacua
- understand H and $H_{(2)} \leftrightarrow$ orbifold geometry

Thank you

The scalar potential

$$V(T,S) = e^{\mathscr{K}}\left(\mathscr{K}^{S\bar{S}}F_S\bar{F}_{\bar{S}} + \mathscr{K}^{T\bar{T}}F_T\bar{F}_{\bar{T}} - 3W\bar{W}\right)$$
$$= e^{k(S,\bar{S})}\,Z(T,\bar{T})\,|\Omega(S)|^2\left[\left(A(S,\bar{S})-3\right)|H(T)|^2 + \widehat{V}(T,\bar{T})\right]$$

$$A(S,\bar{S}) = \frac{k^{S\bar{S}}F_S\bar{F}_{\bar{S}}}{|W|^2} = \frac{k^{S\bar{S}}|\Omega_S + K_S\Omega|^2}{|\Omega|^2}$$

$$\widehat{V}(T,\bar{T}) = \frac{-(T-\bar{T})^2}{3}\left|H_T(T) - \frac{3i}{2\pi}H(T)\,\widehat{G}_2(T,\bar{T})\right|^2$$

$$Z(T,\bar{T}) = \frac{1}{i(T-\bar{T})^3|\eta(T)|^{12}}$$

The EFT: gaugino condensation

$$H_{(2)} = \left(\frac{\mathscr{E}_4^3}{\chi_{12}}\right)^n \left(\frac{\mathscr{E}_6^2}{\chi_{12}}\right)^m \left(\frac{\mathscr{E}_4^2\mathscr{E}_6\chi_{10}}{\chi_{12}^2}\right)^\ell \mathscr{P}(j_{(2)}) \longrightarrow H = \left(\frac{E_4}{\eta^8}\right)^n \left(\frac{E_6}{\eta^{12}}\right)^m \mathscr{P}(j)$$

The check:

1) Expansion in iU

$$\mathscr{E}_4 = E_4(T) + 240E_{4,1}(T,V)e^{-2\pi U} + \ldots$$

$$\mathscr{E}_6 = E_6(T) - 504E_{6,1}(T,V)e^{-2\pi U} + \ldots$$

$$\chi_{10} = \phi_{10,1}(T,V)e^{-2\pi U} + \ldots$$

$$\chi_{12} = \eta^{24}(T) + \frac{1}{12}\phi_{12,1}(T,V)e^{-2\pi U} + \ldots$$

2) Send $V \to 0$

$$E_{4,1}(T,V) \to E_4(T)$$

$$E_{6,1}(T,V) \to E_6(T)$$

$$\phi_{10,1}(T,V) = \frac{1}{144}(E_6E_{4,1} - E_4E_{6,1}) \to 0$$

$$\phi_{12,1}(T,V) = \frac{1}{144}(E_4^2E_{4,1} - E_6E_{6,1}) \to 12\,\eta^{24}(T)$$