

# Critical Aspects of Moduli Fixing



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Michael Ratz



May 17 2024



Modular Invariance Approach to the Lepton and Quark Flavour Problems: from Bottom-up to Top-down, MITP, Mainz, Germany

Based on:

V. Knapp-Pérez, X.-G. Liu, H.P. Nilles, S. Ramos-Sánchez & M.R., Phys.Lett.B 844 (2023) 138106


Outline

&

Disclaimers

# Before we start...

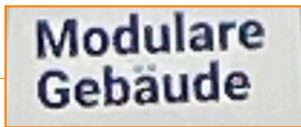
## Apologies and disclaimers:

- 🙄 This talk will not have extensive references to all activities that contribute to this exciting field, sorry!
- 🔗 The  symbols provide you with links to the respective references.
- 🙄 I will suppress many details which are not essential to get the big picture.

# Philosophy of this talk



# Philosophy of this talk



# Philosophy of this talk



Modulare  
Gebäude

||

Modular  
Building

# Philosophy of this talk 😊



Modulare  
Gebäude

||  
Modular  
Building

=

Modular  
Model  
Building  
without  
Model(s)

# Modular flavor symmetries

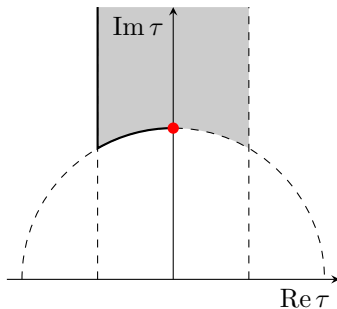
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- In many phenomenologically viable scenarios the best-fit values of  $\tau$  occur close to, but slightly away from, the critical points  $\tau = i$  or  $\tau = \omega := e^{2\pi i/3}$

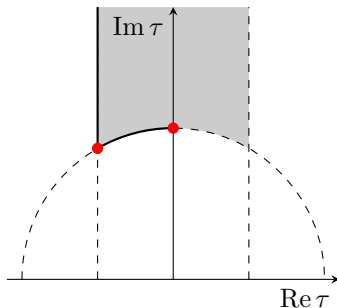
talks by [Gui-Jun Ding](#), [João Penedo](#) & [Morimitsu Tanimoto](#)



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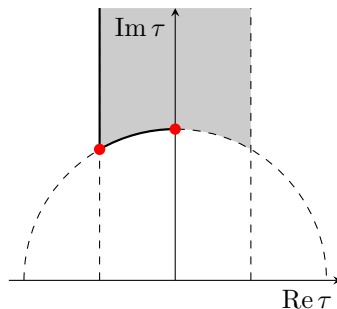
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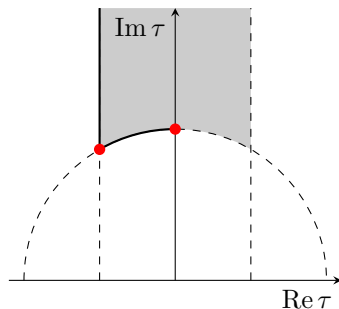
? Can these values be explained dynamically?

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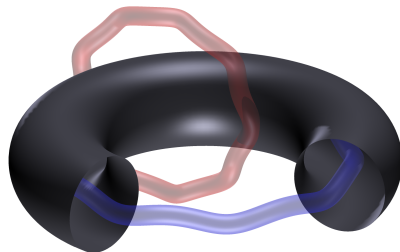
- Challenge: find mechanisms that explain values of  $\tau$  close to but not precisely at the critical points

# String compactifications



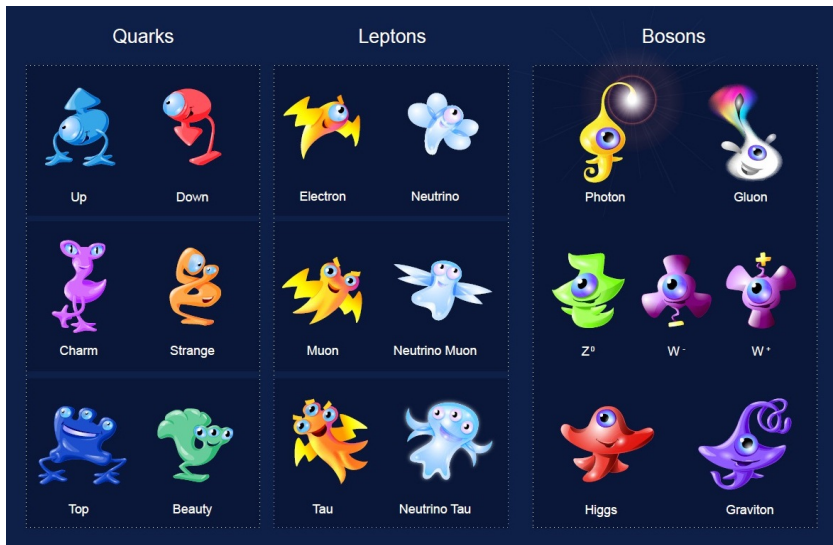
- ☞ Violins: need to be built in such a way that the oscillating strings produce the right sounds

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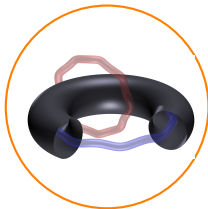


- ☞ Violins: need to be built in such a way that the oscillating strings produce the right sounds
- ☞ String compactifications: twist the strings in such a way that the excitations carry the quantum numbers of the standard model particles

# String phenomenology

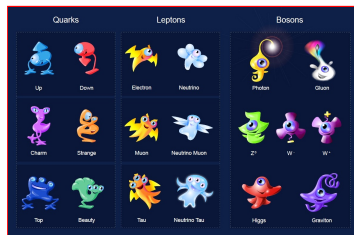
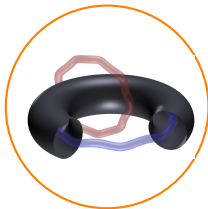


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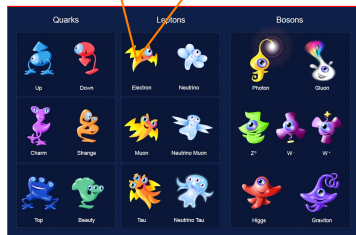
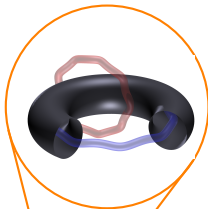




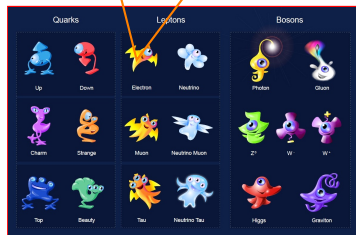
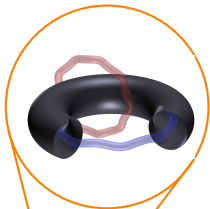
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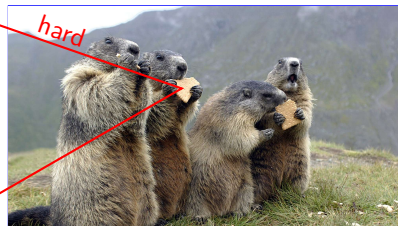
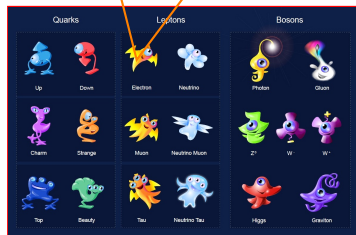
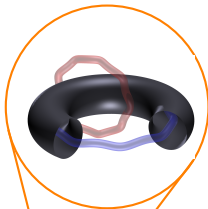
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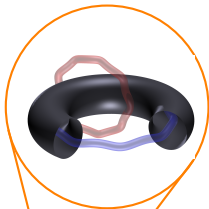
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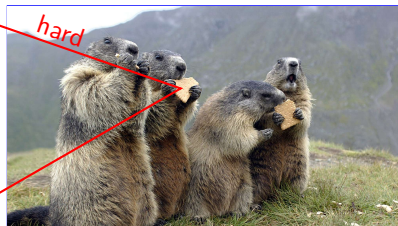
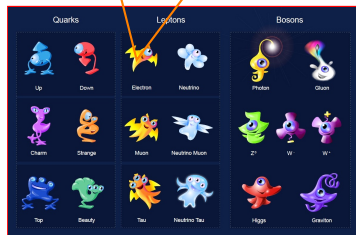
# String phenomenology



# String phenomenology



not easy either



Moduli Fixing

Moduli Fixing

in

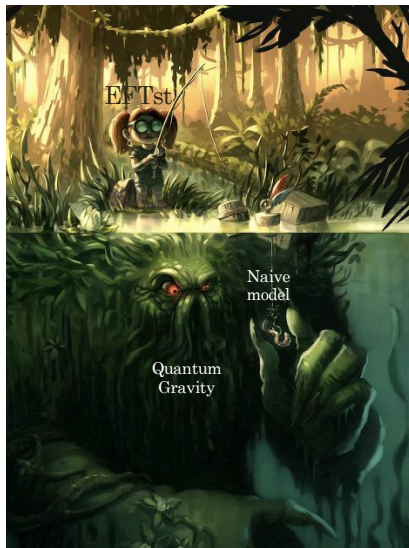
Realistic

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String Compactifications

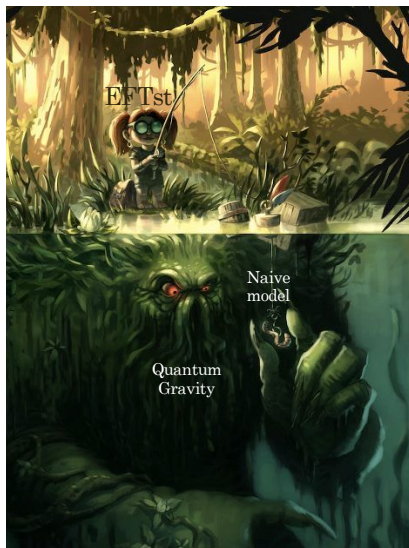
String Compactifications

# Strings and the real world



[https://workshops.ift.uam-csic.es/uploads/poster/poster\\_congreso\\_299.pdf](https://workshops.ift.uam-csic.es/uploads/poster/poster_congreso_299.pdf)

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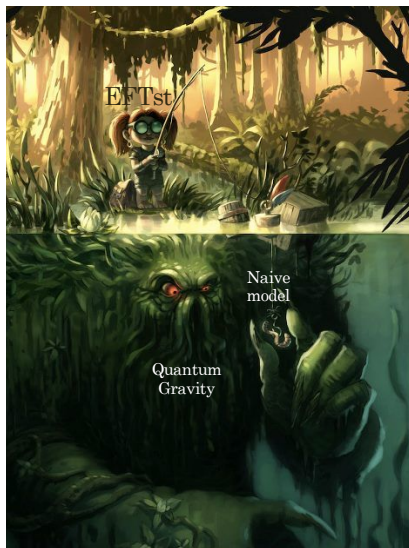


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- 👉 Naive bottom-up models failing to fulfill consistency conditions are ruled out. . .
- . . . and so are top-down models which are inconsistent with observation

# Particle physics from heterotic orbifolds

- 👉 Heterotic orbifolds have a long and rich history in providing us with potentially realistic models

🔗 [Ibáñez, Kim, Nilles & Quevedo \[1987\]](#); ...; 🔗 [Ramos-Sánchez & MR \[2024\]](#)

as opposed  
to unrealistic

substantial  
literature

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- 👉 Explicit MSSM models with the following ingredients exist:
  - exactly 3 generations (vector-like exotics can be consistently decoupled)
  - no (fractionally charged or other) exotics

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$R$  symmetries



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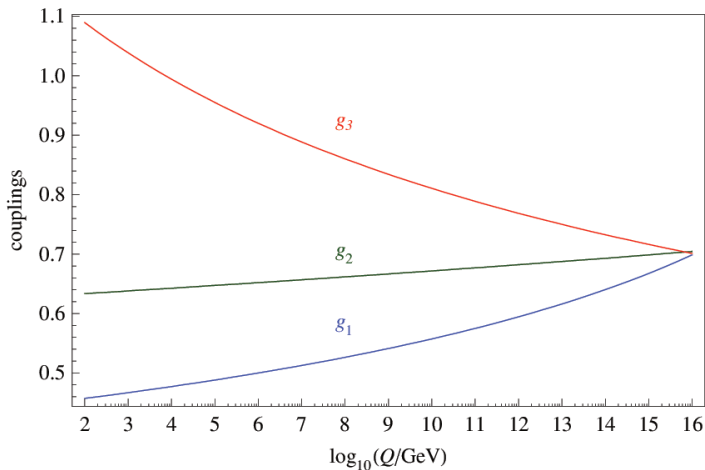
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- ☞ In order to make these statements we need to make sure that the compact dimensions are stabilized at appropriate sizes

# Appropriate compactification radii

cf. e.g. [Dienes \[1997\]](#)

⚡ tension between  $M_{\text{GUT}} \simeq \text{few} \cdot 10^{16} \text{ GeV}$  and typical compactification radii



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[Witten \[1996\]](#):...

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string tension

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- ☞ In practice even this is too hard
- ➔ Let's just toy with one geometric modulus plus dilaton

e.g. [Brandenberger & Vafa \[1989\]](#)

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[Witten \[1981\]](#);...

$$\Lambda \sim M_{\text{P}} \exp(-b/g^2)$$

RG invariant scale

$\mathcal{O}(1)$  coupling

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$$\Lambda \sim M_{\text{P}} \exp(-b/g^2)$$

- hierarchically small gravitino mass

[Nilles \[1982\];...](#)

$$m_{\text{W}} \sim m_{3/2} \sim \frac{\Lambda^3}{M_{\text{P}}^2}$$

... provided that SUSY is broken

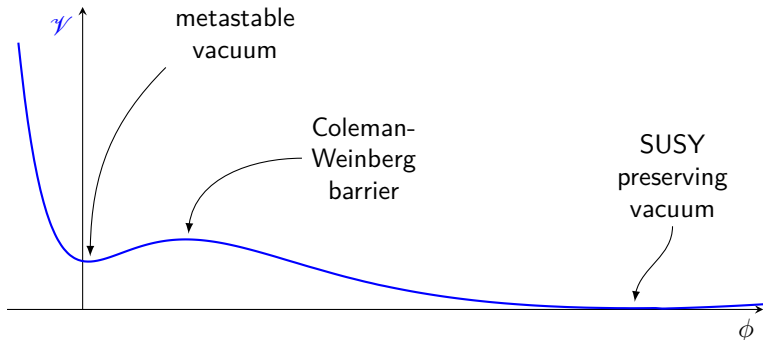
# Breakdown of global SUSY

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see e.g. [✎ Shadmi & Shirman \[2000\]](#)

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[Intriligator, Seiberg & Shih \[2006\]](#);...

- Effective description around metastable minimum:

[Intriligator & Seiberg \[2007\]](#)

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local description  
of CW potential

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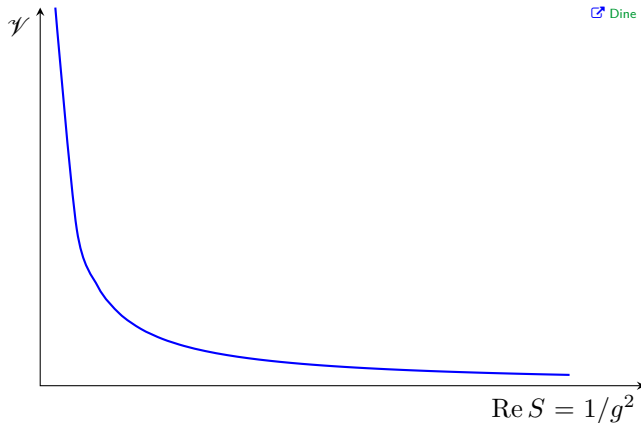
... works for a fixed gauge coupling...

# Problem with string theory embedding

cf. talk by [Nicole Righi](#)

👉 **However:** embedding into string theory  $\leadsto$  run-away problem

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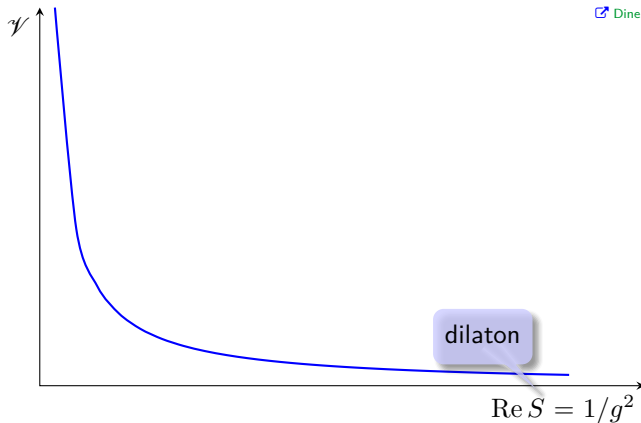


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# Constant + exponential scheme

[Kachru, Kallosh, Linde & Trivedi \[2003\]](#)

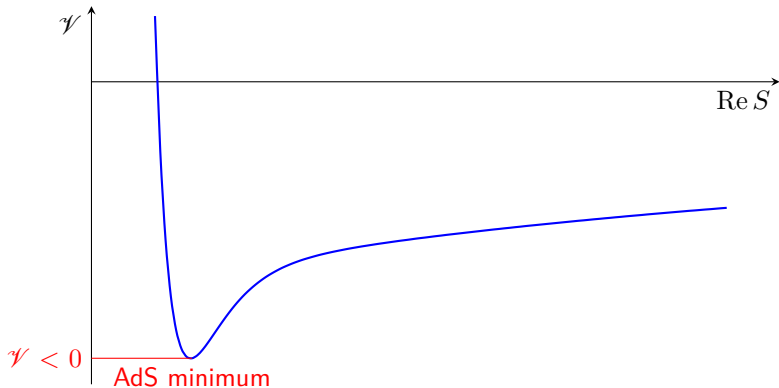
 KKLT type proposal:
 
$$\begin{cases} K_{\text{toy KKLT}} = -\ln(S + \bar{S}) \\ \mathcal{W}_{\text{toy KKLT}} = c - B e^{-bS} \end{cases}$$

constant

non-perturbative

# Constant + exponential scheme

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$$\begin{cases} K_{\text{toy KKLT}} = -\ln(S + \bar{S}) \\ \mathcal{W}_{\text{toy KKLT}} = c - B e^{-bS} \end{cases}$$
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# Constant + exponential scheme

[Kachru, Kallosh, Linde & Trivedi \[2003\]](#)

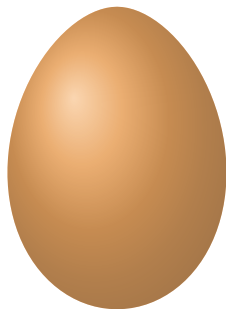
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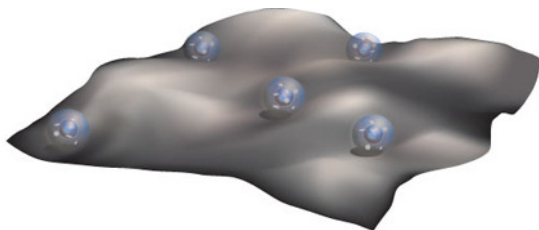
👉 Gravitino mass:  $m_{3/2} \sim c \xrightarrow{m_{3/2} \stackrel{!}{\simeq} \text{TeV}} |c| \sim 10^{-15}$



Chicken or egg problem

# Constant + exponential scheme

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# Constant + exponential scheme

[Kachru, Kallosh, Linde & Trivedi \[2003\]](#)

KKLT type proposal: 
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Gravitino mass:  $m_{3/2} \sim c \xrightarrow{m_{3/2} \stackrel{!}{\sim} \text{TeV}} |c| \sim 10^{-15}$

Philosophy of flux compactifications: many vacua, in some of them  $c$  might be small by accident

Alternative proposal: hierarchically small expectation of the perturbative superpotential due to an approximate  $U(1)_R$  symmetry

[Kappl, Nilles, Ramos-Sánchez, MR, Schmidt-Hoberg & Vaudrevange \[2009\]](#)

$$c \rightarrow \langle \mathcal{W}_{\text{pert}} \rangle \sim \langle \phi \rangle^N$$

typical VEV  $\ll M_P$

$N = \mathcal{O}(10)$   
order of  
explicit  
 $U(1)_R$  breaking

# FI terms and breakdown of modular invariance

- The overwhelming majority of potentially realistic heterotic orbifold models has a Fayet–Iliopoulos (FI)  $D$ -term

$$\xi_{\text{FI}} = \frac{\text{tr } q_{\text{anom}}}{192\pi^2} M_{\text{P}}^2$$



# FI terms and breakdown of modular invariance

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chiral superfield  
with nontrivial  $U(1)$   
charges

- This does not signal an inconsistency but simply means that some “matter” fields have to acquire VEVs of the order  $v_{\text{Cabibbo}} M_{\text{P}}$

🔗 [Atick, Dixon & Sen \[1987\]](#)

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- This gives rise to Froggatt–Nielsen type scenarios

[Binétry, Lavignac & Ramond \[1996\]](#)

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[Atick, Dixon & Sen \[1987\]](#)

- This gives rise to Froggatt–Nielsen type scenarios

[Binétruy, Lavignac & Ramond \[1996\]](#)

- Given that virtually all “matter” fields carry some nontrivial modular weights, this means that modular symmetry is broken spontaneously by the VEVs needed to cancel the FI term

# Putting things together



- 👉 The  $\text{exp} + \text{constant}$  scheme allows us to fix the dilaton in an  $\text{adS}$  supersymmetric vacuum

[Kachru, Kallosh, Linde & Trivedi \[2003\]:...](#)

# Putting things together

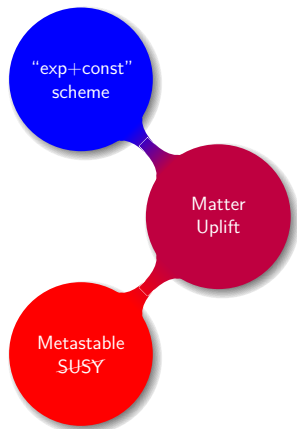
"exp+const"  
scheme

Metastable  
SUSY

- ☞ Metastable SUSY breaking yields a positive contribution to the vacuum energy

[☞ Intriligator, Seiberg & Shih \[2006\]:...](#)

# Putting things together



👉 Uplifting via matter fields leads to a distinctive pattern of soft masses

[Lebedev, Nilles & MR \[2006\]:...](#)

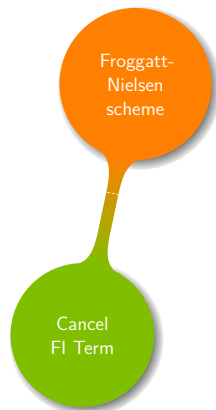
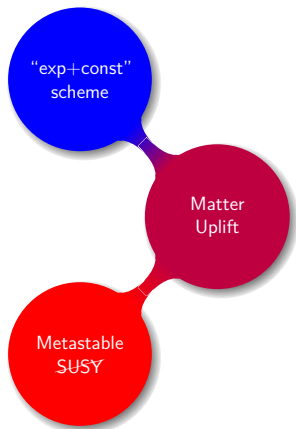
# Putting things together



☞ The FI term induces VEVs of matter fields



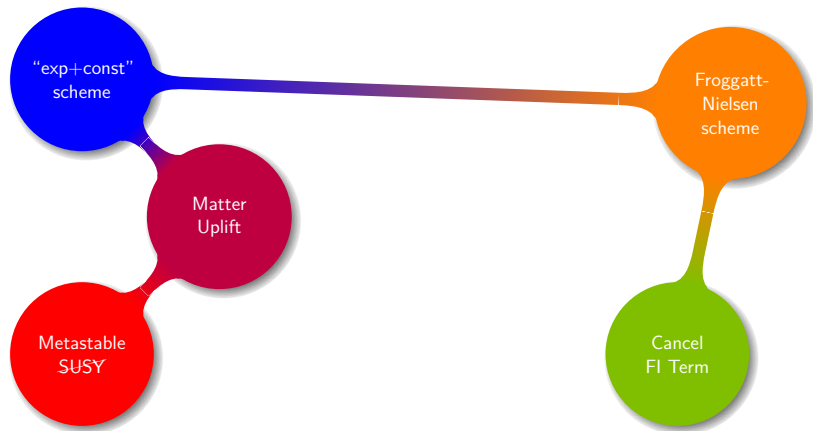
# Putting things together



☞ Among other things this leads to flavor hierarchies a la Froggatt-Nielsen. . .

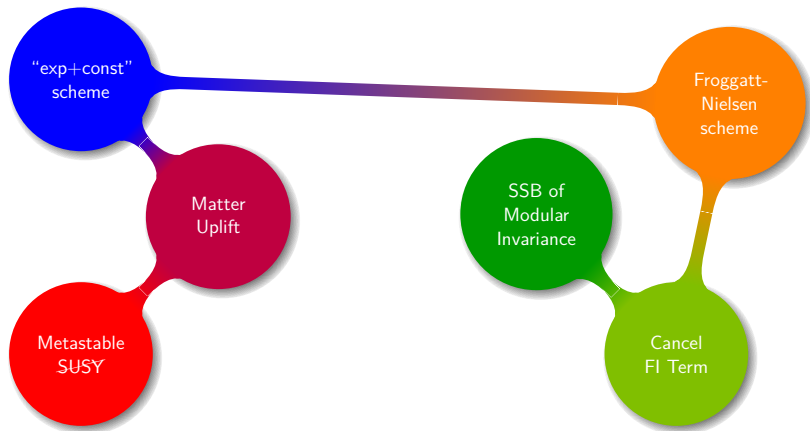
[Binétruy, Lavignac & Ramond \[1996\]:...](#)

# Putting things together



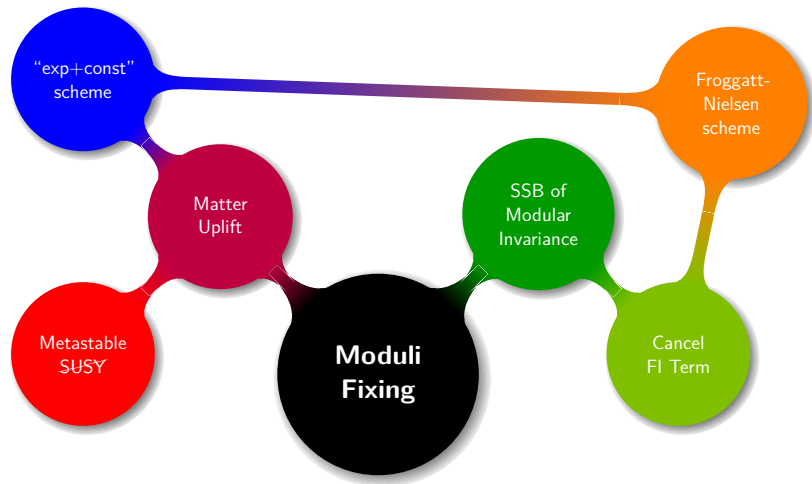
- 👉 Moderate hierarchy gets enhanced by approximate  $R$  symmetries to yield a tiny constant [Kappl, Nilles, Ramos-Sánchez, MR, Schmidt-Hoberg & Vaudrevange \[2009\];...](#)

# Putting things together



- ☞ ... and it leads to a spontaneous breakdown of modular invariance by matter field VEVs

# Putting things together



- ☞ As we shall see, these effects can move the modulus  $\tau$  slightly away from the symmetry enhanced points

# Modular invariant gaugino condensation

[Font, Ibáñez, Lüst & Quevedo \[1990\]](#); [Nilles & Olechowski \[1990\]](#); [Cvetič, Font, Ibanez, Lüst & Quevedo \[1991\]](#);...

➡ Modular covariant superpotential describing gaugino condensate

$$\mathcal{W}_{\text{gc}}(S, \tau) = \frac{\Omega(S) H(\tau)}{\eta^6(\tau)}$$

Dedekind  
 $\eta$ -function

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- Ingredients

$$\Omega(S) = B e^{-bS}$$

$\beta$ -function  
coefficient

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[Font, Ibáñez, Lüst & Quevedo \[1990\]](#); [Nilles & Olechowski \[1990\]](#); [Cvetič, Font, Ibanez, Lüst & Quevedo \[1991\]](#);...

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$$\Omega(S) = B e^{-bS}$$

$$H(\tau) = \left( \frac{E_4(\tau)}{\eta^8(\tau)} \right)^n \left( \frac{E_6(\tau)}{\eta^{12}(\tau)} \right)^m P(j(\tau))$$

Eisenstein  
series

integers

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Klein  $j$ -function

polynomial



# Modular invariant gaugino condensation

[Font, Ibáñez, Lüst & Quevedo \[1990\]](#); [Nilles & Olechowski \[1990\]](#); [Cvetič, Font, Ibanez, Lüst & Quevedo \[1991\]](#);...

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[Dent \[2002\]](#); [Kobayashi, Shimizu, Takagi, Tanimoto & Tatsuishi \[2019\]](#); [Ishiguro, Kobayashi & Otsuka \[2021\]](#)  
[Novichkov, Penedo & Petcov \[2022\]](#); [Ishiguro, Okada & Otsuka \[2022\]](#); [Leedom, Righi & Westphal \[2023\]](#); talk by **Nicole Righi**

☞ Vacua away from the critical lines

☞ Uplifting via Shenker terms

[Leedom, Righi & Westphal \[2023\]](#); talk by **Nicole Righi**

# Involving matter fields

## ☞ Kähler and superpotential

☞ Knapp-Perez, Liu, Nilles, Ramos-Sánchez & MR [2023]

$$K = -\ln(S + \bar{S}) - 3\ln(-i\tau + i\bar{\tau})$$

$$+ (-i\tau + i\bar{\tau})^{-k_X} \bar{X}X - (-i\tau + i\bar{\tau})^{-2k_X} \frac{(\bar{X}X)^2}{\Lambda_{\text{ISS}}^2}$$

ISS field

local description  
of Coleman–Weinberg  
potential

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 \end{aligned}$$

small constants  
induced by  
matter fields  
of various  
modular weights

gaugino  
condensate

effective  
description  
of ISS sector

Fixing  $\tau$  close to  $i$ 

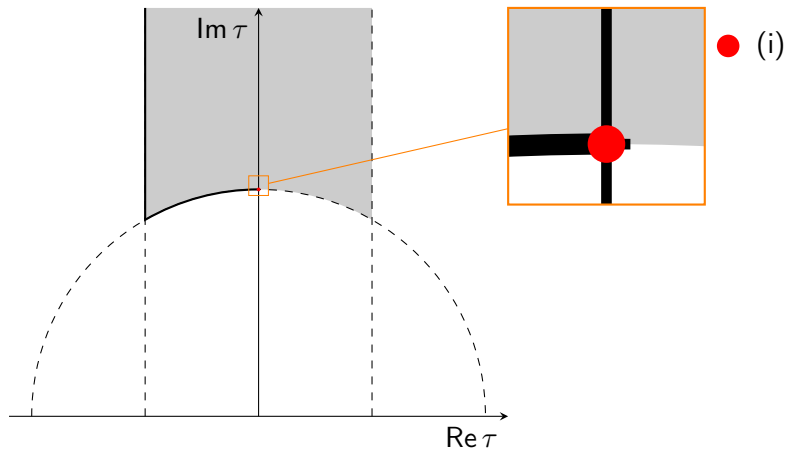
[Knapp-Perez, Liu, Nilles, Ramos-Sánchez & MR \[2023\]](#)

$$(m, n) = (0, 1), \quad c_1 = 2 \cdot 10^{-8}, \quad (k_c, k_X) = (1, 0)$$

$$b = 10, \quad B = 1, \quad \Lambda_{\text{ISS}} = 10^{-9}$$

$c_2$	$f_X$	$k_Y$	$\tau$	$\langle \mathcal{V} \rangle$
<b>0</b>	<b>0</b>	<b>0</b>	<b><math>i</math></b>	<b><math>&lt; 0</math></b>
$2e^{i\pi/3} \cdot 10^{-8}$	0	0	$-0.014 + 1.015i$	$< 0$
<b>0</b>	<b><math>3.49 \cdot 10^{-8}</math></b>	<b>0</b>	<b><math>i</math></b>	<b><math>\simeq 0</math></b>
0	$6 \cdot 10^{-8}$	1	$1.010i$	$\simeq 0$
$2e^{i\pi/3} \cdot 10^{-8}$	$5 \cdot 10^{-8}$	0	$-0.018 + 1.011i$	$\simeq 0$
$2e^{i\pi/3} \cdot 10^{-8}$	$8.5 \cdot 10^{-8}$	1	$-0.018 + 1.021i$	$\simeq 0$

# Stabilization of $\tau$



👉 No matter fields, only  $\tau$ , with or without uplift

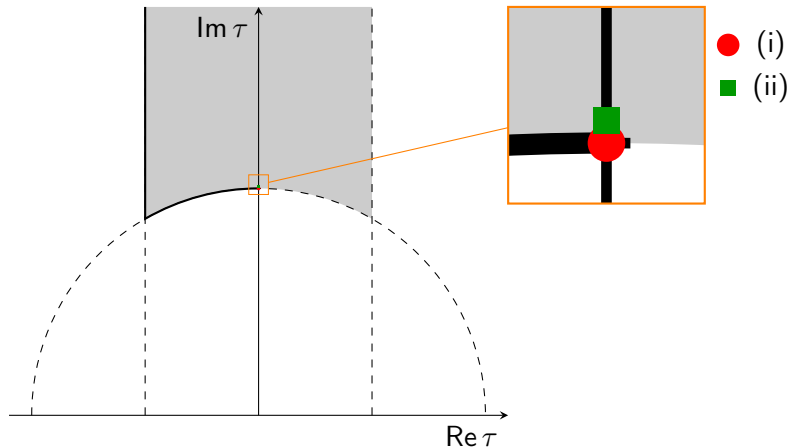
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[Knapp-Perez, Liu, Nilles, Ramos-Sánchez & MR \[2023\]](#)

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# Stabilization of $\tau$



👉 Breaking of modular invariance by matter fields

Fixing  $\tau$  close to  $i$ 

[Knapp-Perez, Liu, Nilles, Ramos-Sánchez & MR \[2023\]](#)

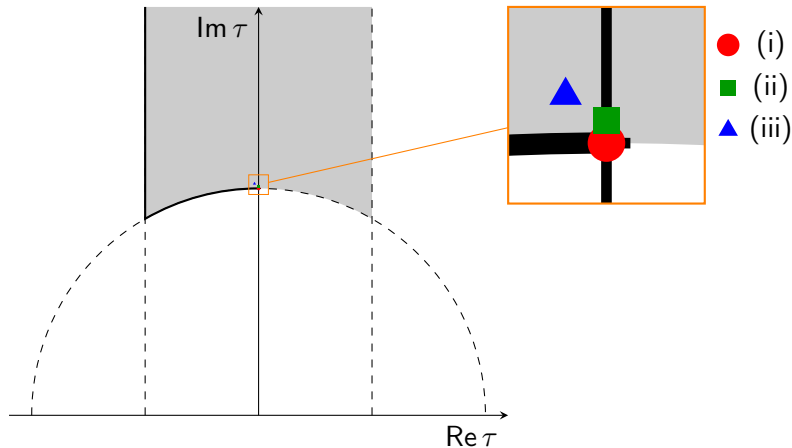
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# Stabilization of $\tau$



👉 Spontaneous breakdown of modular invariance by matter field and uplift

# Fixing $\tau$ close to $\omega := e^{2\pi i/3}$

[Knapp-Perez, Liu, Nilles, Ramos-Sánchez & MR \[2023\]](#)

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$$K = -\ln(S + \bar{S}) - 3\ln(-i\tau + i\bar{\tau})$$

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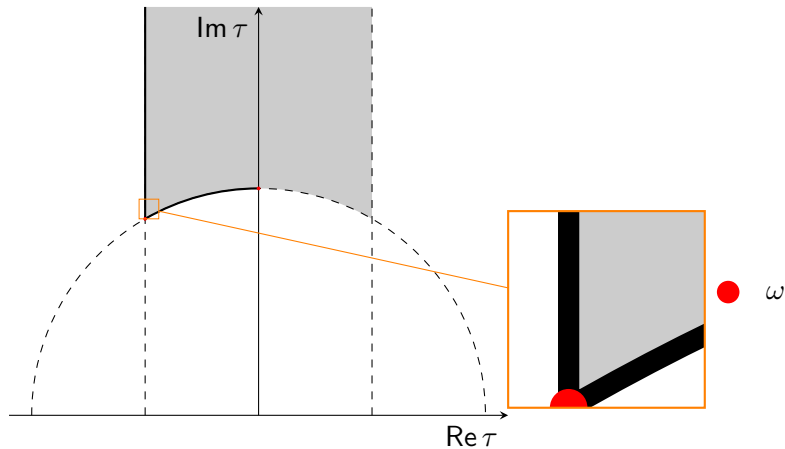
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$$(m, n) = (1, 0), \quad c_1 = 2 \cdot 10^{-8}, \quad c_2 = 0, \quad b = 10, \quad B = 1, \quad k_c = 0, \quad f_X = 0$$

fixes

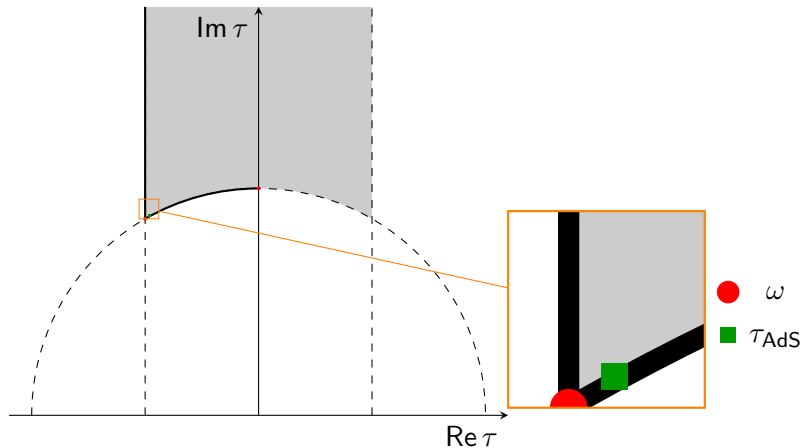
$$\tau_{\text{AdS}} \simeq -0.48 + 0.88i, \quad S \simeq 2.15 \quad \text{and} \quad \mathcal{V} \simeq -1.28 \cdot 10^{-12} < 0$$

# Stabilization of $\tau$



👁 No matter fields, only  $\tau$

# Stabilization of $\tau$



Spontaneous breakdown of modular invariance by matter field

# Fixing $\tau$ close to $\omega$

[Knapp-Perez, Liu, Nilles, Ramos-Sánchez & MR \[2023\]](#)

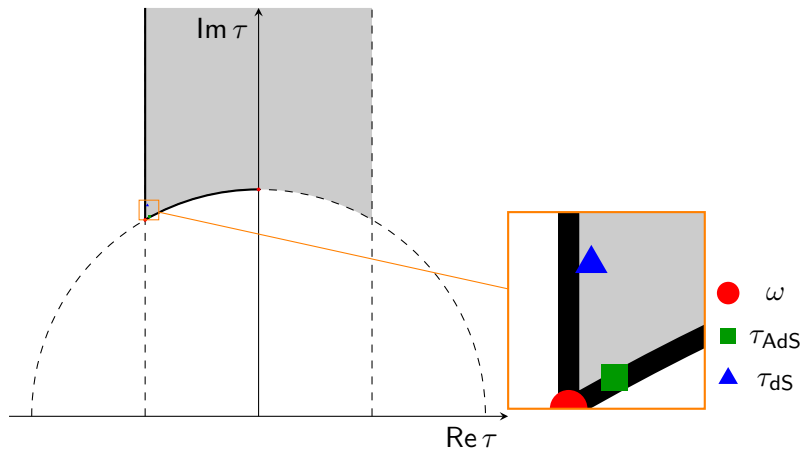
$$(m, n) = (1, 0), \quad c_1 = 2 \cdot 10^{-8}, \quad c_2 = 0, \quad b = 10, \quad B = 1, \quad k_c = 0$$

$$\Lambda_{\text{ISS}} = 10^{-7}, \quad k_X = 0, \quad k_Y = 1 \quad \text{and} \quad f_X \simeq 6 \cdot 10^{-8}$$

fixes

$$\tau_{\text{dS}} \simeq -0.49 + 0.94i \quad \text{and} \quad S \simeq 2.16$$

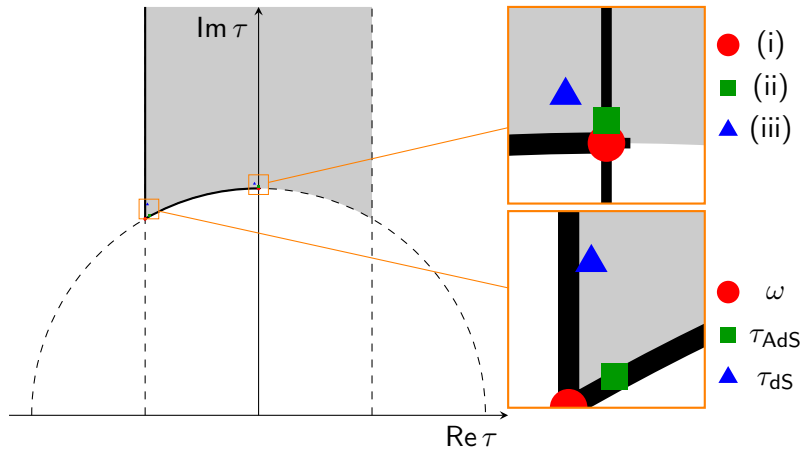
# Stabilization of $\tau$




Spontaneous breakdown of modular invariance by matter field and uplift



# Stabilization of $\tau$



 Survey of all cases

Summary

Summary

&

Outlook

Outlook

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- Delicate interplay between perturbative and nonperturbative effects can fix the dilaton and the modulus  $\tau$

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# Summary

- ✚ Delicate interplay between perturbative and nonperturbative effects can fix the dilaton and the modulus  $\tau$
- ✚ Crucial ingredients:
  - ① spontaneous breakdown of modular invariance when fields of nontrivial modular weights attain VEV
  - ② positive contributions to the vacuum energy from (metastable) dynamical supersymmetry breaking
- ✚ Spontaneous breaking of modular invariance by matter fields unavoidable in explicit string models:
  - ① cancellation of FI term
  - ② breaking of extra gauge symmetries

# Outlook



👉 Loose ends:

- 1 explicit stabilization of matter field VEVs

# Outlook



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- 1 explicit stabilization of matter field VEVs
- 2 enhanced symmetries at critical points



# Outlook



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# Outlook



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  - ② enhanced symmetries at critical points
  - ③ multiple moduli
- 👉 Further insights from top-down models
- 👉 More precision

Thank you very much!

Υμιν ευχαριστώ πολύ!



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The image features a red background with a spotlight effect on the character. The character is a cartoon man with a white cap, glasses, a blue vest, and grey pants, holding a bouquet of blue and white flowers. The word 'DANKESCHÖN' is written in large white letters at the top. The 'MAINZEL MÄNNCHEN' logo is in the bottom left, and the website 'mainzelmaennchen.zdf.de' is in the bottom right. A vertical URL 'www.zdf-werbefernsehen.de' is on the far right.

Backup slides

Backup slides

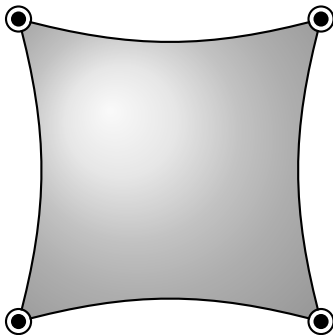
# $R$ symmetries

- Discrete  $R$  symmetries arise as remnants of the Lorentz symmetry of compact dimensions and are arguably on the same footing as the fundamental symmetries  $\mathcal{C}$ ,  $\mathcal{P}$  and  $\mathcal{T}$

[▶ back](#)

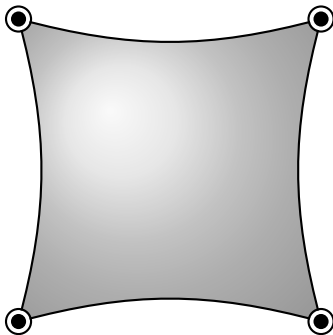
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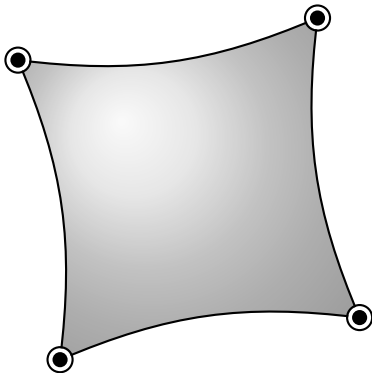
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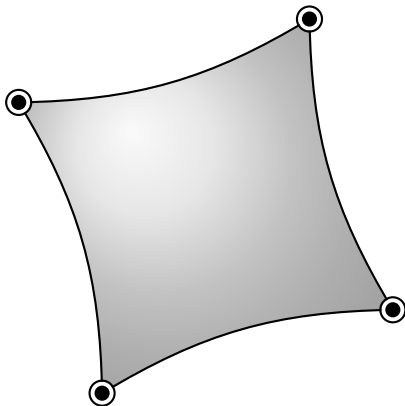
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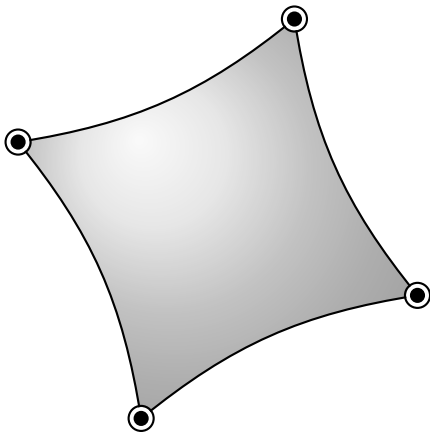
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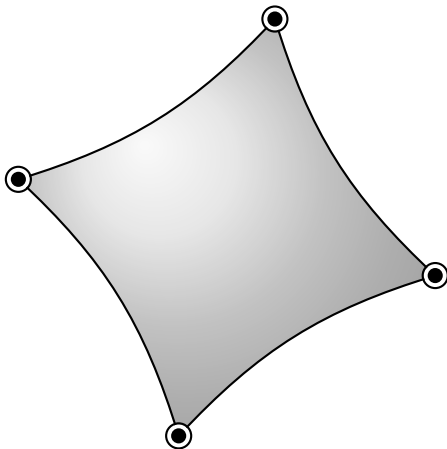
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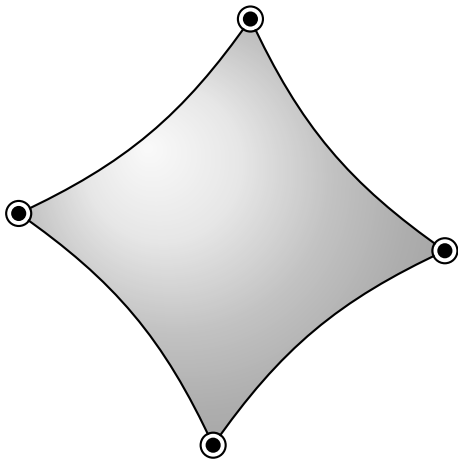
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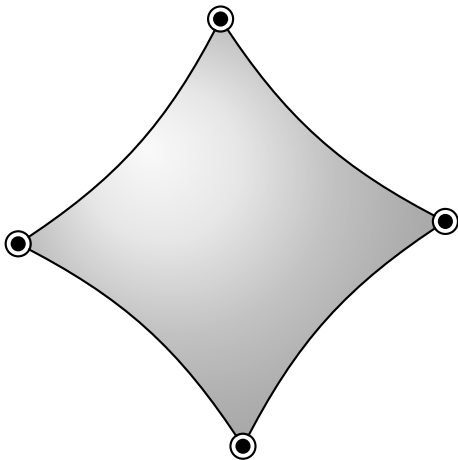
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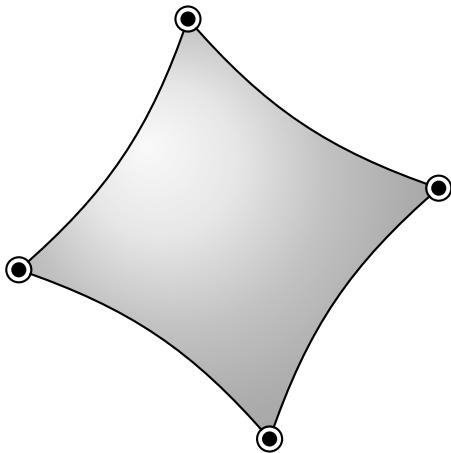
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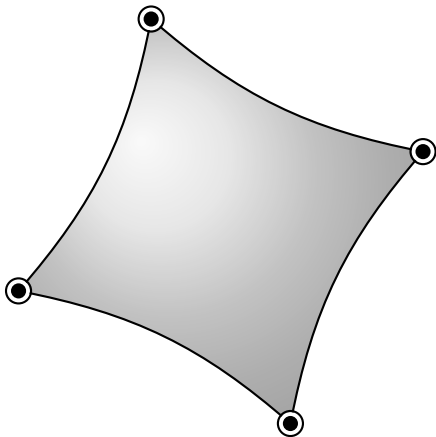
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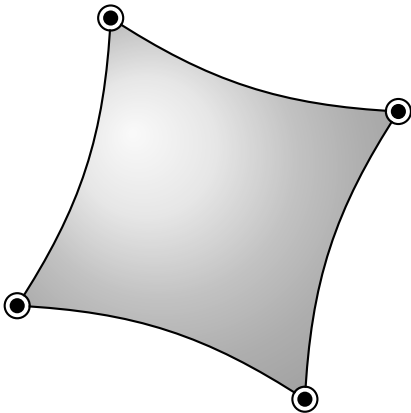
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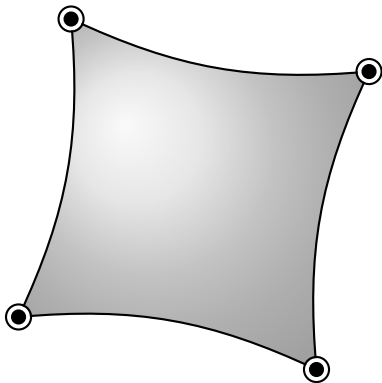
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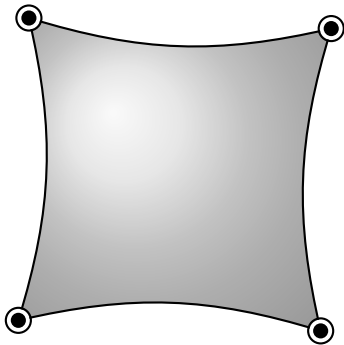
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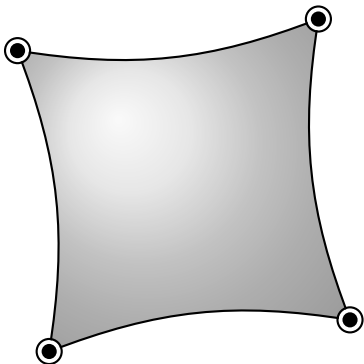
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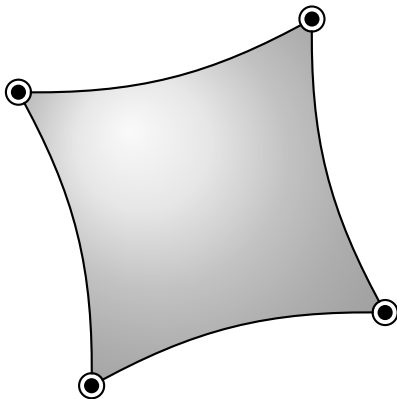
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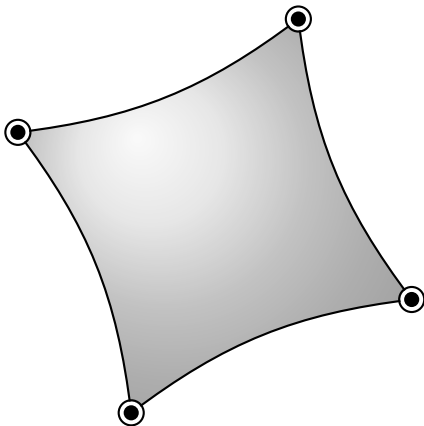
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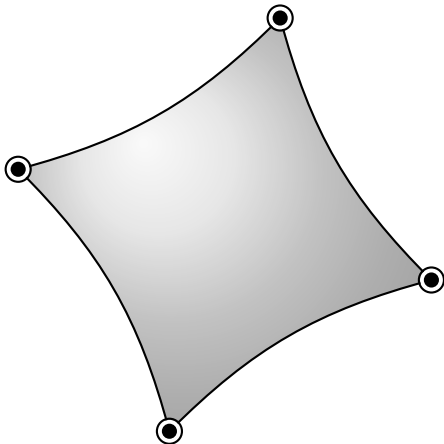
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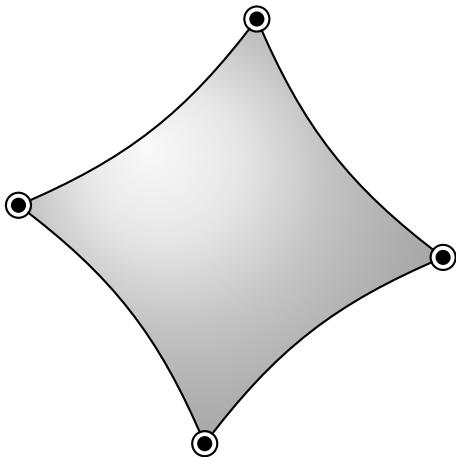
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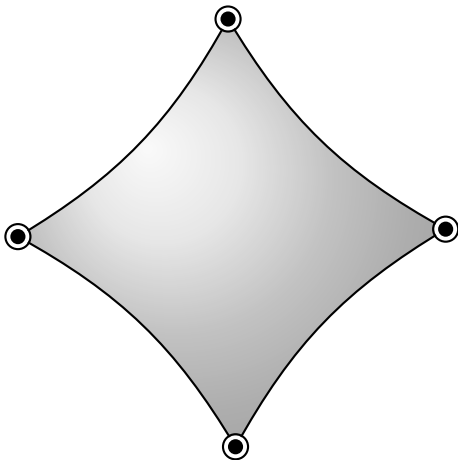
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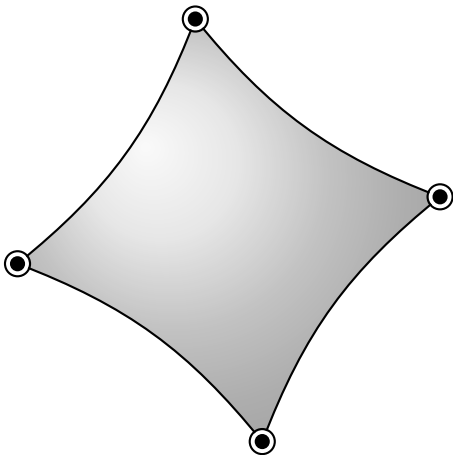
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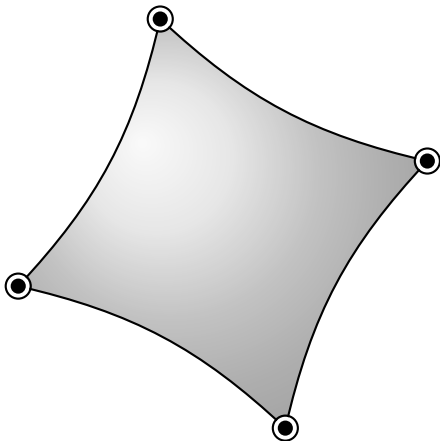
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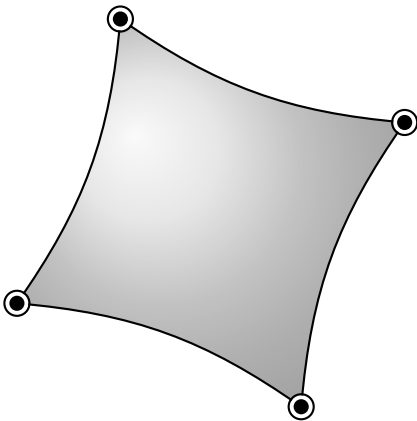
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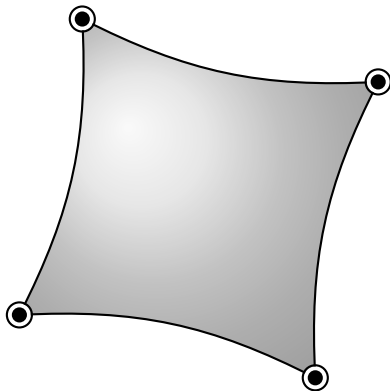
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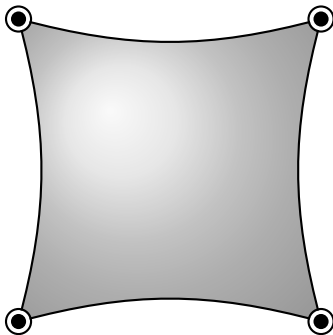
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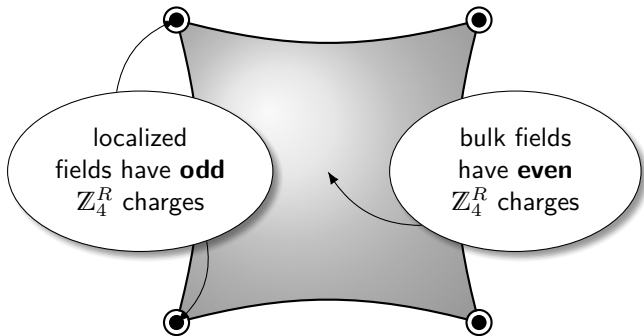
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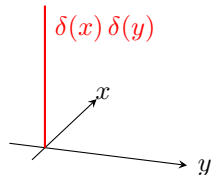
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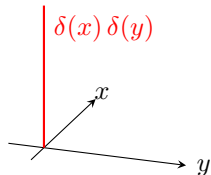
# Interlude: Field theory vs. string theory



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- In string theory it follows from  $H$ -momentum conservation that localized (twisted) states have odd  $\mathbb{Z}_4^R$  charges while bulk (untwisted) have even  $\mathbb{Z}_4^R$  charges

# Hierarchically small $\langle \mathcal{W} \rangle$

Two observations:

- 1 in the presence of an exact  $U(1)_R$  symmetry

$$\frac{\partial \mathcal{W}}{\partial \phi_i} = 0 \quad \curvearrowright \quad \langle \mathcal{W} \rangle = 0$$

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fields

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- 2 for approximate  $R$  symmetries

$$\langle \mathcal{W} \rangle \sim \langle \phi \rangle^N$$

typical  
field VEV

order  
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$$\langle \mathcal{W} \rangle = 0 \text{ because of } U(1)_R \quad (I)$$

**aim:** show that

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Consider a superpotential

$$\mathcal{W} = \sum c_{n_1 \dots n_M} \phi_1^{n_1} \dots \phi_M^{n_M}$$

with an exact  $R$  symmetry

$$\mathcal{W} \mapsto e^{2i\alpha} \mathcal{W}, \quad \phi_j \mapsto \phi'_j = e^{i r_j \alpha} \phi_j$$

where each monomial in  $\mathcal{W}$  has total  $R$  charge 2

$$\langle \mathcal{W} \rangle = 0 \text{ because of } U(1)_R \quad (\text{II})$$

Consider a field configuration  $\langle \phi_i \rangle$  with

$$F_i = \frac{\partial \mathcal{W}}{\partial \phi_i} = 0 \quad \text{at } \phi_j = \langle \phi_j \rangle$$

Under an infinitesimal  $U(1)_R$  transformation, the superpotential transforms nontrivially

$$\mathcal{W}(\phi_j) \mapsto \mathcal{W}(\phi'_j) = \mathcal{W}(\phi_j) + \sum_i \frac{\partial \mathcal{W}}{\partial \phi_i} \Delta \phi_i$$

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This is only possible if  $\langle \mathcal{W} \rangle = 0!$

**bottom-line:**

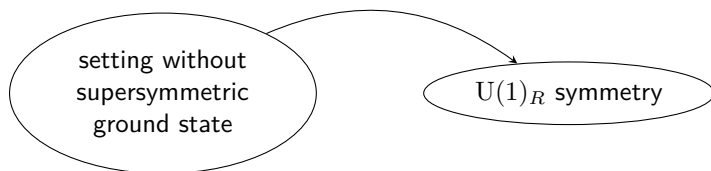
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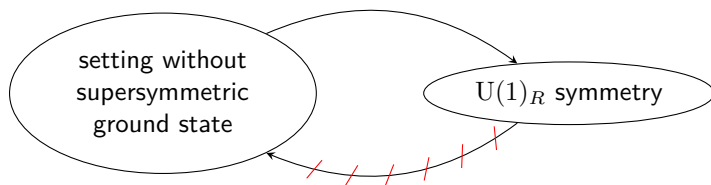
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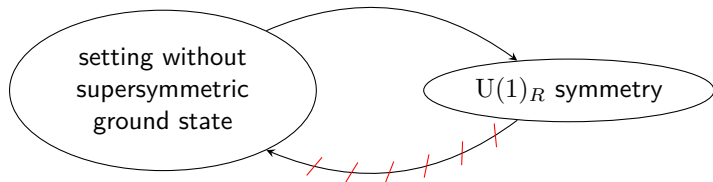
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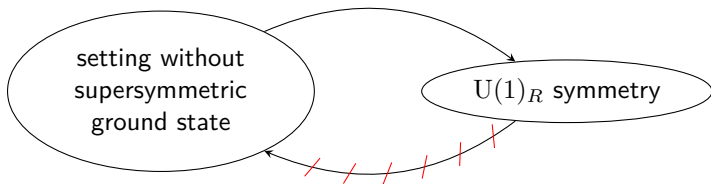
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 (That is, a  $U(1)_R$  symmetry implies Minkowski solutions.)

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- 4 in 'no-scale' type settings

solutions of global  
SUSY  $F$  term eq.'s

$=$

stationary points of super-  
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[Weinberg \[1989\]](#)

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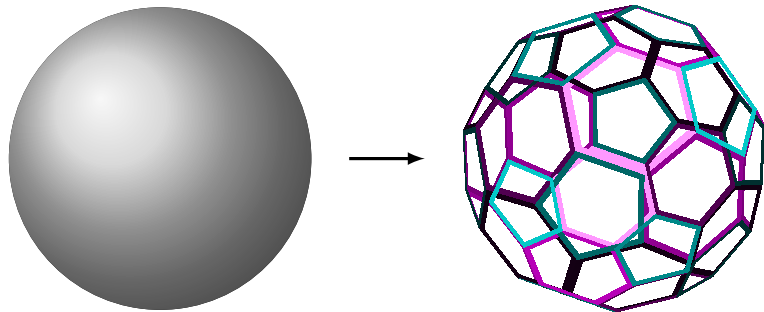
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- ☞ Such an **approximate**  $U(1)_R$  symmetries can be a consequence of discrete  $\mathbb{Z}_N^R$  symmetries
- ☞ Confirmed in various field-theoretic examples

Explicit  
string theory  
realization

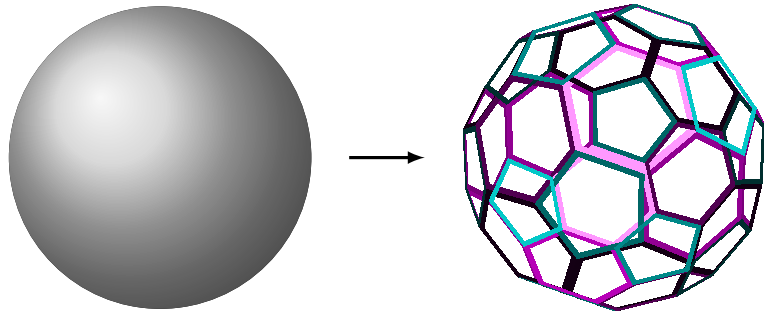
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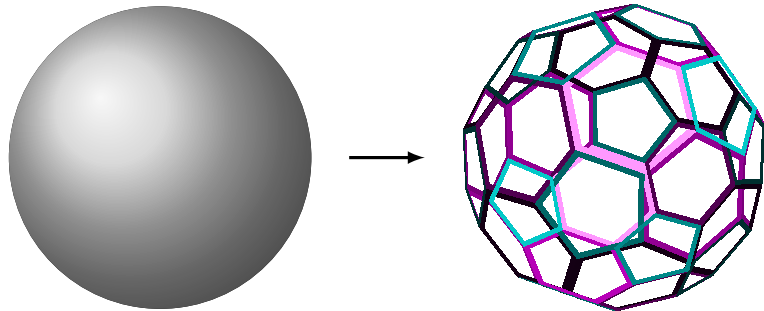
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- For example: a  $\mathbb{Z}_2$  orbifold plane leads to  $\mathbb{Z}_4^R$  symmetry

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- Studied the previous example ('heterotic benchmark model IA') with 23 SM singlets  $s_i$  getting a VEV



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**Note:** in order to prove the existence a full understanding of coupling coefficients is required

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## bottom-line:

straightforward embedding in heterotic orbifolds

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- ☞ The more fields are switched on, the lower  $N$  we obtain examples:
- model with 23 fields  $\curvearrowright N = 9$
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- 👉 Minima survive supergravity corrections

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